Efficient Use of Bandwidth in the Wideband Regime

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Determining the infinite-bandwidth capacity is equivalent to finding the minimum energy required to transmit one bit of information reliably. To obtain this quantity, we can choose to maximize the information per unit energy in contrast to the standard Shannon setting in which the information per degree of freedom is maximized. Motivated by the optimality of on-off signaling in the infinite bandwidth limit, Gallager [1], found the exact reliability function in the setting of a binary-input channel where information is normalized, not to blocklength, but to the number of '1's contained in the codeword. More generally, we can pose the "capacity per unit cost" problem where an arbitrary cost function is defined on the input alphabet [2]. An important class of cost functions are those which, like energy, assign a zero cost to one of the input symbols. For those cost functions, the capacity per unit cost not only is equal to the derivative at zero cost of the Shannon capacity but admits a simple formula [2]. Even in this more general setting, capacity per unit cost is achieved with on-off signaling with vanishing duty cycle.

A wide variety of digital communication systems (particularly in wireless, satellite, deep-space, and sensor networks) operate in the power-limited region where both spectral efficiency (b/s/Hz) and energy-per-bit are relatively low. The information theoretic analysis of those channels, in addition to to leading to the most efficient bandwidth utilization, reveals design insights on good signaling strategies.

The following conclusions about signaling and capacity in the wideband regime have been drawn in the literature:

- On-off signaling approaches capacity as the duty cycle vanishes.
- The derivative at zero signal-to-noise ratio of the Shannon capacity determines the wideband fundamental limits.
- Capacity is not affected by fading.
- Receiver knowledge of channel fade coefficients is useless.

In this presentation, we show that these conclusions are misguided.

In addition to transmitter/receiver complexity, attaining the minimum energy per bit entails zero spectral efficiency. A communication engineer who needs to transmit a given data rate through a given available bandwidth, and wants to find the required power does not find the solution in $\frac{E_b}{N_0}_{\min}$ nor on the derivative at zero signal-to-noise ratio of the Shannon capacity. Of course, the solution lies in the full Shannon capacity function for arbitrary signal-to-noise ratios. Unfortunately, the capacity function and the inputs that attain it are unknown for many channels of interest, particularly in the presence of fading. However, as we show in this presentation, it is possible to obtain analytically the fundamental limits of a general class of additive-noise channels in the wideband regime in which the spectral efficiency is small but nonzero. These results offer engineering guidance on the fundamental bandwidth-power tradeoff and on signaling strategies that attain it in the wideband limit.

The tradeoff power vs. bandwidth is mirrored in the tradeoff of the information theoretic quantities spectral efficiency vs. E_b/N_0 (energy per bit normalized to background noise spectral level). Our approach for the wideband regime is to approximate spectral efficiency as an affine function of E_b/N_0 (dB). Thus, we are interested in obtaining not only $\frac{E_b}{N_0}_{\min}$ but the *wideband slope* of the spectral efficiency-$\frac{E_b}{N_0}$ curve in b/s/Hz/(3 dB) at $\frac{E_b}{N_0}_{\min}$. Spectral efficiency in the wideband regime turns out to be determined by both the first and second derivatives of the channel capacity at zero signal-to-noise ratio.

We establish in considerably wider generality than was previously known, that the received energy per bit normalized to noise spectral level in a Gaussian channel subject to fading is -1.59 dB, regardless of side information at the transmitter and/or receiver. On-off signaling for the unfaded white Gaussian noise channel turns out to require more than six times the bandwidth required by QPSK, which is shown to be wideband optimal in both $\frac{E_b}{N_0}_{\min}$ and b/s/Hz/(3 dB). Receiver knowledge of fading coefficients is shown to have a deep impact on both the required bandwidth and the optimal signaling strategies. When the channel has an unknown component, approaching $\frac{E_b}{N_0}_{\min}$ turns out to be very demanding both in bandwidth and in the peak-to-average ratio of the transmitted signals. The kurtosis of the fading distribution is shown to play a key role in determining the required bandwidth when the receiver is able to track the channel coefficients.

References

[1] R. G. Gallager, "Power limited channels: Coding, multiaccess, and spread spectrum," *1988 Conf. on Information Science and Systems*, Mar 1988.

[2] S. Verdú, "On channel capacity per unit cost," *IEEE Trans. Information Theory*, vol. 36 (5), pp. 1019–1030, Sep. 1990.