Classification of Correlated Flat Fading MIMO Channels (Multiple Antenna Channels)

Zachary Bagley and Christian Schlegel
Department of Electrical Engineering
University of Utah
Salt Lake City, UT 84112

Email: schlegel@ee.utah.edu
zbagley@csw.l-3com.com

1 Introduction

It is well known that the use of multiple transmit and receive antennas has the potential to increase the capacity of a wireless link by a factor as high as the minimum of the numbers of transmit and receive antennas [2, 1]. However, these capacity calculations assume that the fading statistics between any antenna pair is independent from the statistics between any other pair.

In real world situations, component distortions, insufficient separation between array elements, and, in particular, realistic scattering environments cause the fading statistics between antenna elements to be correlated among the elements, which contradicts the assumption of independent channel path gains often seen in the literature.

This paper studies such multiple-input multiple-output (MIMO) channels in terms of their capacity based upon simple, but basic assumptions on the scattering environment.

2 Channel Model

The channel model assumes t transmit and r receive antennas. Each antenna transmits n symbols from a symbol alphabet (M-PSK, FSK, QAM, etc.), each with power ρ/t per signaling interval. The receiver decodes the received signal based on the sampled noisy outputs of the r transmit antennas. The channel is modeled as a $r \times t$ matrix $\mathbf{H}$ where h_{ij} is the path gain from the i^{th} transmit antenna to the j^{th} receive antenna. As a mobile receiver moves through a wave front, it experiences fading on each path h_{ij}.

The received signal $y_n^{(j)}$ for the j^{th} receive antenna at time n is then given by

$$y_n^{(j)} = \sum_{i=1}^{T} h_{ij} c_{in} \sqrt{\rho} + \eta_{jn} \tag{1}$$

or in matrix form

$$\mathbf{Y} = \sqrt{\rho}\mathbf{HC} + \mathbf{N} \tag{2}$$

The noise is modeled as complex Gaussian with variance of N_0 per complex dimension.

The information theoretic capacity of such a channel is given by [2]

$$C = E_{\mathbf{H}}(C_I) \tag{3}$$

and

$$C_I = \log\left(\mathbf{I} + \frac{\rho}{t}\mathbf{HH}^*\right) \tag{4}$$

can be interpreted as the "instantaneous" channel capacity for a given channel realization $\mathbf{H}$ and total received power per element, ρ. An example of a channel with a rich transmitter local scattering environment is shown in the figure below. The left-hand plot shows the received signal magnitude, and the upper plot shows the instantaneous capacity for a 2×2 system with several wavelengths of separation between antenna array elements.

The symbol frequency is $f_s = 1$ MHz, which for the particular model geometry ensures that no intersymbol interference is generated.

The speed of the mobile array is $v = 50$ kph. As a consequence very rapid fluctuations of the channel state occur, arguably the largest challenge to applying multi-antenna techniques to mobile communications.

3 Channel Emulation

We model the transmission environment with a simple core equation for the flat fading channel emulator based on ray tracing techniques. Emulation of the channel is done by finding the multi-path delay and Doppler frequency profiles given a signaling interval, rate of motion, and the number of scattering objects.

If the symbol time $T_s = 1/f_s$ is chosen such that frequency non-selective fading occurs, the multi-path channel between the i^{th} transmitter element and the j^{th} re-

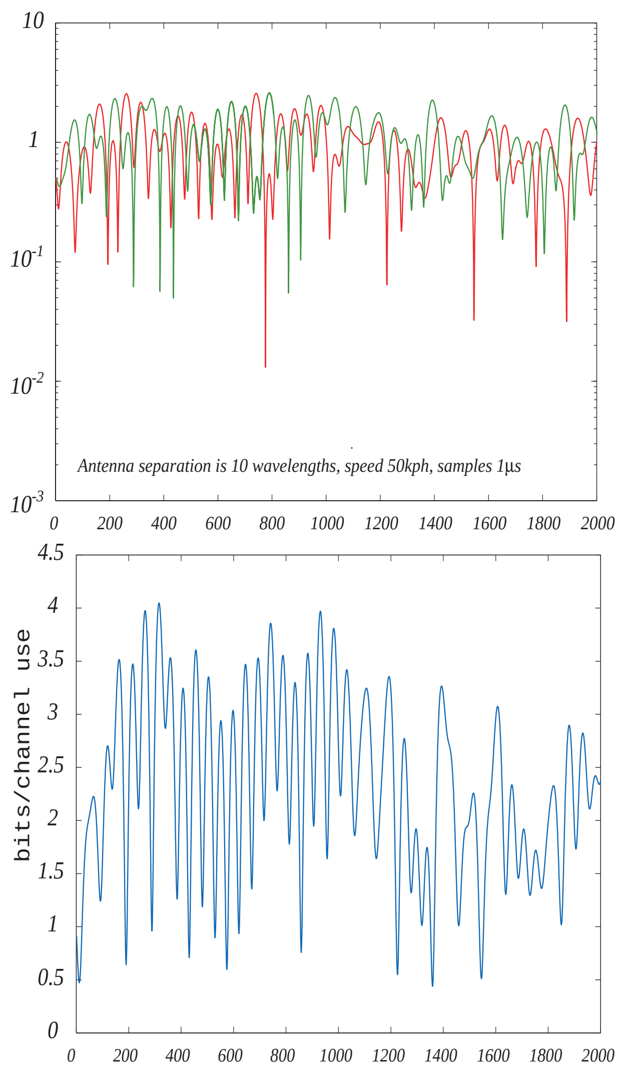

Figure 1: Magnitude response and capacity for a 2×2 system at 0dB

ceiver element may be represented by a single time varying complex scalar value given by

$$h_{ij}(n) = A \sum_{m=1}^{M} g_m e^{-j(\theta_m + 2\pi f_{d,m} nTs)} \tag{5}$$

where M is the number of rays and A is a normalization constant so that each h_{ij} has a unit variance. The random variables g_m, θ_m, and $f_{d,m}$ are the amplitude, phase, and Doppler frequency, respectively, of the m^{th} path between the i^{th} transmitter and the j^{th} receiver array elements.

While this is a simple model it captures the essential issues involving MUA-communication, in particular the effect of correlation and clutter location on the instantaneous and average channel capacities.

Three scenarios are examined. Case (a) has clutter around the mobile antenna array, Case (b) has clutter around the base-station antenna array, and Case (c) is the worst case where the distance between mobile and base is very large compared to the radii of the clutter distributions, leading to completely correlated channels.

3.1 Case (a): Clutter around Mobile

Assuming the objects are somewhat uniformly distributed around the mobile, the spatial angles of arrivals are widely distributed, causing a delay profile of several wavelengths. Movement causes rather rapid fluctuation in the received signals as shown in Figure 1

However having many antennas tends to average this fading effect, reducing both second and third moments of C_I, as seen in Figure 2.

The physical measurements with a 10 antenna system in a rich local scattering environment made by Swindlehurst, et. al, [5] have shown the capacity to be essentially unaffected with array orientation and location, as our channel emulation also indicates.

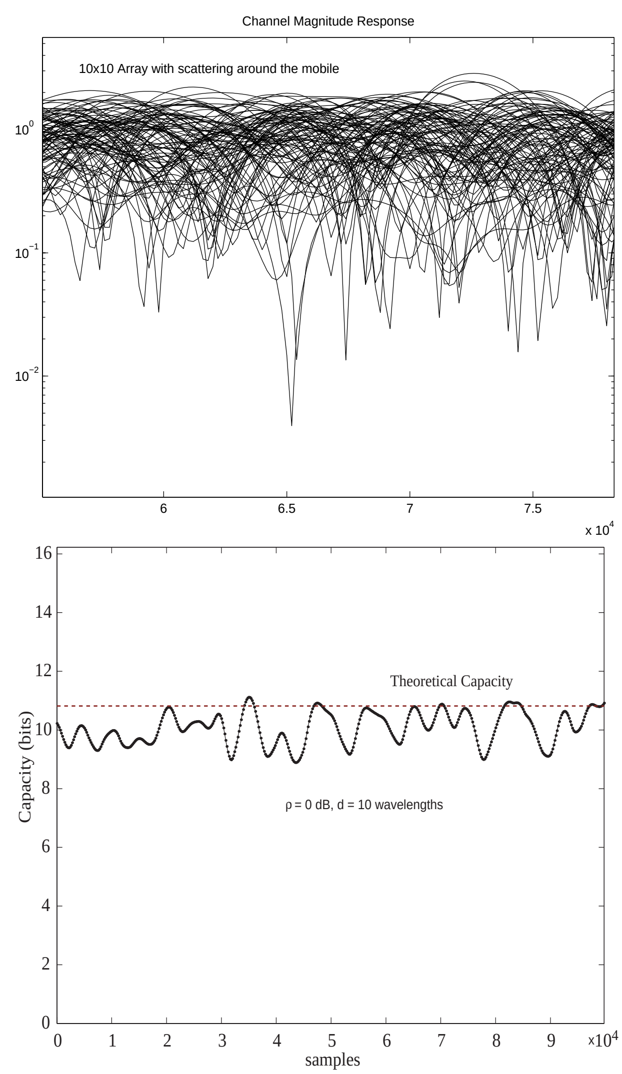

Figure 2: 10×10 array with clutter around mobile

3.2 Case (b): Clutter around Base

When the wavefronts are reflected off stationary objects near a stationary array, the only mechanism for generating a Doppler profile with non-zero bandwidth is the moving array. The spatial angles relative to the moving array can be very small depending on the location of the scattering objects. The Doppler profile bandwidth is smaller than for Case (a), but the channel path gains are more correlated, as shown in Figure 3 for a 5×5 system. Even though the channel is highly correlated, it is not completely correlated and so a MUA system will still have higher capacity than a single antenna system with the same amount of power.

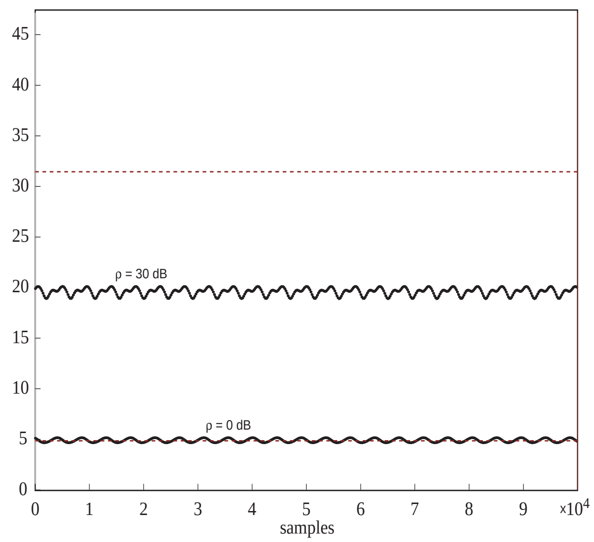

Figure 3: 5×5 array with clutter around base.

3.3 Case (c): Distance Arrays

For the case of antenna arrays separated by a large distance compared to the radius of the clutter distribution, the system essentially becomes a line-of-sight channel with all the paths highly correlated. The transmitter and receiver arrays are still creating many paths with slightly different pathlengths determined by the antenna spacing, but all paths are correlated, causing $\mathbf{HH}^*$ to have only one non-zero eigenvalue for any reasonable antenna spacing.

4 Effect of Correlation

We now examine how correlation in the channel effects capacity. Let λ_i be the i^{th} eigenvalue of $\mathbf{HH}^*$ and μ be a constant chosen to meet the transmitted power constraint of

$$\rho \leq tr(\mathbf{xx}^*) \tag{6}$$

as given in [2]. The "instantaneous" power and capacity expressions from [2] are

$$P(\mu) = \sum_i \max(0, \mu - \lambda_i^{-1}), \tag{7}$$

$$C(\mu) = \sum_i \max(0, \log(\mu\lambda_i)) \tag{8}$$

with the complex noise power, N_0, set to unity.

When the channel is uncorrelated the capacity has the general behavior

$$C \approx \min(r,t)\log(1 + \mu\lambda_i). \tag{9}$$

For the completely correlated case, there is only one non-zero eigenvalue, λ_i. In this case, the capacity is roughly

$$C \approx \log(1 + \sum_i \mu\lambda_i) \approx \log(1 + \min(r,t)\mu\lambda_i) \tag{10}$$

and we see that in a completely correlated channel the capacity grows logarithmically, rather than linearly with the number of antennas.

When the SNR is low, a Taylor Series approximation shows $\log(1+x) \approx x$ for small x, making the capacity of the independent channel case given by

$$C \approx \min(r,t)\log(1 + \mu\lambda_i) \approx \min(r,t)\mu\lambda_i. \tag{11}$$

When the channel is correlated we have

$$C \approx \log(1 + \min(r,t)\mu\lambda_i) \approx \min(r,t)\mu\lambda_i \tag{12}$$

which indicates correlation in the channel has no effect on capacity when the SNR is very low, and an increased number of antennas has the sole effect of gathering more power. Communications is power limited and the additional dimensions afforded by multiple antennas can not be expoited.

4.1 Number and Spacing of Antennas

Antenna spacing directly affects channel correlation and therefore channel capacity as seen in Figure 4 for a 4×4 array at 0 and 20dB values for ρ. In general, the antenna spacing requirement for independent channels is reduced as the number of scattered rays increase. However in extreme cases such as the satellite channel or low SNR, even very large antenna separations will not produce independent channels. For Case (c) this is due to the very small difference in the incident angles of the wavefronts arriving at the scattering objects.

When the multipath environment is rich around the antenna array and the SNR is high, the variance decreases and mean increases linearly with the number of antennas. This is clearly the best case scenario. A rich multipath environment is generated with only a few scattering objects when the number of rays is large. The Law of Large Numbers applies to the number of rays $r \times t$, making each of the channel matrix elements tend toward a complex Gaussian distribution with only a few antennas and only a few scattering objects.

When the scattering environment is located around the base, the channel correlation properties are dominated by the location of the scattering objects. If the objects are uniformly distributed around the base array, the capacity is essentially the same as in case (a), just with different fading characteristics. This is illustrated in 4. If the objects are all located close to one another, the receiver array sees only small path length differences and the situation tends toward that of case (c).

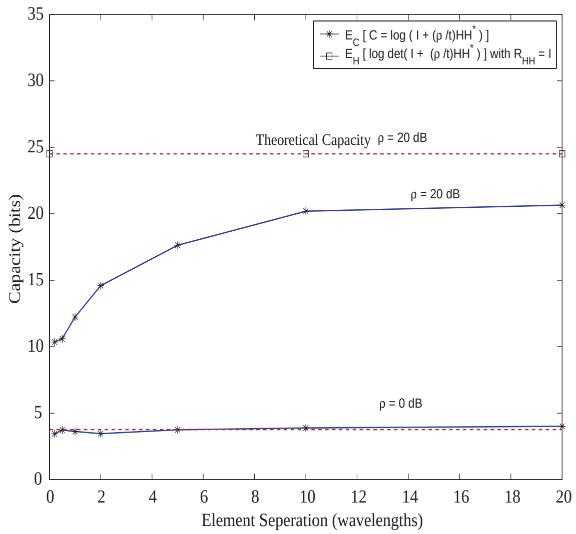

Figure 4: Average capacity of 4×4 array

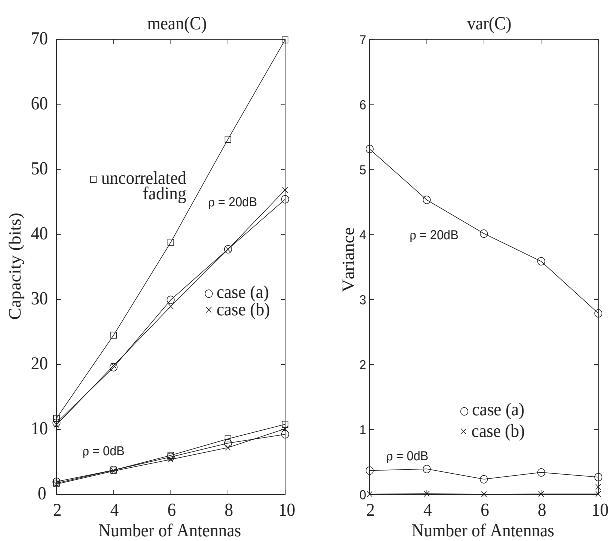

Figure 5: Mean and Variance versus N

The plots of Figures 4 and 5 had 15 scattering objects uniformly distributed inside a ring of 50m with a distance of 2km separating the arrays. The carrier frequency is 2.4GHz and the symbol time is 1 MHz.

5 Conclusion

We have seen that for high signal-to-noise ratios the multiple antenna channel always provides higher capacity than a single antenna channel when the eigenvalues of $\mathbf{HH}^*$ are valued such that the parallel Gaussian channels in the water-filling solution for channel capacity carry information under the transmitted power constraint. Correlation in the channel has the effect of reducing the number of effective parallel Gaussian channels. For low signal-to-noise ratios such as 0 dB, the efficient operating point of Turbo-coded applications, the multiple antenna system is power limited, and the correlation of the channels has no effect on capacity.

When the SNR is high, rich local scattering around the mobile causes capacity to fluctuate rapidly around an average value that increases linearly with the number of antennas, but only logarithmically in the SNR. Scattering around the base also increases capacity, but is affected much more by the number and location of the scattering objects. If the distance between the arrays is very large compared to the location of the scattering objects, the channel is essentially completely correlated and capacity increases only logarithmically with the number of receive antennas.

6 Acknowledgments

The partial support of NSF for this project under grant ECS 9979323 and the support of L3 Communications, Salt Lake City, in conjunction with this project is gratefully acknowledged.

References

[1] J.H. Winters, J. Salz, G.D. Gitlin, "The capacity of wireless communication systems can be substantially increased by the use of antenna diversity," *Proc. Conf. Inform. Sci. Syst.*, Princeton, NJ, 1992.

[2] E. Telatar, "Capacity of Multi-antenna Gaussian Channels", *IEEE Trans. Commun.*, vol. 42, No. 2/3/4, pp. 1617–1627, November-December 1999.

[3] C. Schlegel, "Error probability calculation for multibeam Rayleigh channels", *IEEE Trans. Commun.*, vol. 44, No. 3, pp. 290–293, March 1996.

[4] Brian L. Hughes, "Optimal Space-Time Constellations from Groups," *submitted to IEEE Trans. Commun.*, March 2000.

[5] A. Lee Swindlehurst, Gus German, Jon Wallace, Michael Jensen, "Experimental Measurements of Capacity for MIMO Indoor Wireless Channels," *Third IEEE Signal Processing Workshop, Taoyuan, Taiwan*, March 20-23, 2001