Polyphase Realization of VSB Filter Banks for Spectrally Efficient Transmitter Multiplexers and Receiver Demultiplexers Using Spectral Factorization

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***Abstract*-Various Nyquist polynomials and their spectral factorizations to obtain suitable finite impulse response (FIR) vestigial sideband (VSB) filters are summarized here. The filters of interest have transfer functions that are polynomials with complex coefficients which are symmetric, due to Lawton, and polynomials whose coefficients are conjugate symmetric or conjugate antisymmetric, which are also linear phase. The resulting polyphase implementations using polynomials with complex coefficients for the VSB filters result in computational savings of 25% over linear phase prototype VSB filters with real coefficients, and 50% over VSB filters obtained by factorization into minimum and maximum phase polynomials. Relationships to symmetries of the complex scaling functions and wavelets obtained from the usual two-scale difference equations are described.**

***Index Terms*-Spectral factorization, Nyquist half-band filters, complex wavelets, complex FIR filters, complex half-band filters, polyphase implementations, symmetric wavelets, VSB filter designs, linear phase filters, spectrally efficient communication systems.**

I. INTRODUCTION

To identify the problem that originally motivated this paper, we first provide a concise description of a proposed spectrally efficient design for a multiplexer-demultiplexer pair composed of a corresponding synthesis-analysis tree-structured filter bank pair, with its genesis in wavelet packet-based filter bank trees [1], that we have described in detail in [2]. Figs. 1 and 2 show an example of the architecture of the filter bank at the transmitter, and Fig. 3 shows the block diagram of a matching receiver architecture. We next provide summary descriptions of these figures, which are described in detail, including a design example, in [2-4]. Fig. 1 shows the first of four basic 4-input wavelet packet-based synthesis filter banks whose output signals are the inputs signals $X_0(z)$, $X_1(z)$, $X_2(z)$ and $X_3(z)$ to an inverse discrete Fourier transform (IDFT) polyphase synthesis filter bank shown in Fig. 2. The ensemble of input signals into the four channels shown at the left of Fig. 1 consists of binary streams of +1's and –1's. The filters in Fig. 1 consist of lowpass-highpass pairs, which are usually quadrature mirror filters (QMF's). A multiplexer channel is any path from an input to an upsampler, one level at a time, to the root of the tree. The filter pairs at each level are designed, as described in [3], to be identical at each level, but to have transition bands decreasing by a factor of one-half from level to level from the leaves to the root of the tree. The transition bands are designed as in [3] to result in a specified minimum stopband attenuation of the composite magnitude frequency response of the multiplexer channels that is equal to the minimum stopband attenuation of the individual filters in the QMF pairs. The output signals from the multiplexer channels are orthogonal if the filter pairs are QMF's. For the example of Fig. 1 the stopband attenuation of the QMF pairs in [2, 4] was designed to be somewhat more than 40 dB. Using two lowpass filters with rolloffs of 50% and 25% in the QMF's at the leaves of the tree, and at the level next to the root of the tree, respectively, resulted in a 100% rolloff of the multiplexer channel magnitude frequency responses from the leaves to the root of the tree [2, 4]. Because input signals can be fed into the leaves of the tree in Fig. 1 at say, rate r, or into the upsamplers at the next level toward the root of the tree at rate $2r$, or even directly into the root of the tree, at rate $4r$, a bandwidth-on-demand capability is a very desirable feature of this multiplexer design.

As shown in Fig. 2 of the example the output signals from each of four identical tree-structured filter banks, one of which is shown in Fig. 1, provide four real data streams that are used as input signals into an IDFT. The four output signals from the IDFT are filtered by the polyphase components [5] of a prototype FIR VSB filter with complex coefficients, $A(z)$, which, for this example with M = 8, is an 8^{th}-band filter with 25% rolloff [4], obtained from a lowpass filter design $H(z)$ by frequency shifting its response anti-clockwise by $1/M$ = 1/8 of the whole way around the unit circle of the z-plane. Analytically, for this example, $A(z) = H(ze^{-j\pi/4})$. The output signals from the polyphase filters are each upsampled by 4, delayed as shown in Fig. 2, and summed to obtain the output signal $\hat{X}_1(z)$ of the upper filter bank. An identical set of four filter bank trees, one of which is shown in Fig. 1, provides four output signals into another IDFT whose four output signals are fed into four polyphase filters for which the prototype FIR VSB filter is $jz^{-D}A(1/z)$, where the D is calculated to ensure causality [6]. The outputs of the polyphase filters are again delayed and summed to obtain the signal $\hat{X}_2(z)$ of the lower filter bank indicated in Fig. 2. The two signals $\hat{X}_1(z)$ and $\hat{X}_2(z)$ are then added to obtain the complex output signal of the combined filter bank. In the example there are a total of 32 input signal streams and orthogonal multiplexer channels, illustrated by Figs 1 and. 2, providing a multiplexed stream of complex numbers from these channels for transmission over the channel.

Fig. 3 shows a single carrier receiver demultiplexer that is matched to the transmitter multiplexer of Fig. 2. All the filters can be easily derived from those in the transmitter.

The problem that originally motivated the main subject of this paper is the efficient design of the prototype VSB filter for the DFT polyphase synthesis filter bank [5, Fig. 8.21] denoted as $A(z)$ in Fig. 2. The other prototype VSB filter at the transmitter, as shown in Fig. 2, and the two matching prototype VSB filters at the receiver, as shown in Fig. 3, are then easily derived from $A(z)$. In [2] six designs for $A(z)$ are described. These six designs all result from shifting the frequency response of six FIR filters with real coefficients, $H(z)$, whose magnitude frequency response is symmetric about the origin, to a position with centre at normalized frequency $1/M$, where the radian frequency 2π all around the unit circle is normalized to 1, and M is the total number of channels, e.g., M = 8 in Fig. 2. Fig. 4 shows the magnitude frequency response of a design for an $A(z)$ with

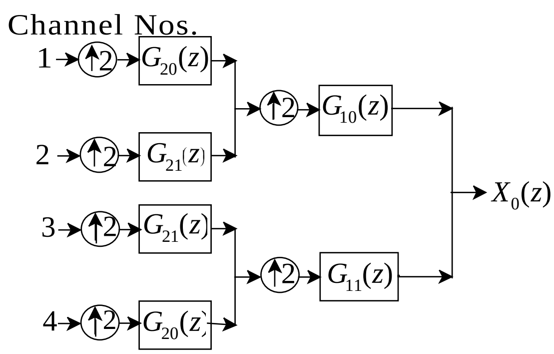


Fig. 1. The first basic 4-input wavelet packet-based synthesis filter bank repeated 3 times, with different channel numbering each time with successive output signals at the roots as the input signals $X_1(z)$, $X_2(z)$ and $X_3(z)$ to a DFT polyphase synthesis filter bank shown in Fig. 2.

equiripple stopband attenuation of over 40 dB that was obtained from frequency shifting the response of a minimum phase $H(z)$ with real coefficients that was obtained by spectral factorization of a polynomial representing a Nyquist filter. The frequency shift of the magnitude frequency response of $H(z)$, which is symmetric about the origin, by $1/M$ = 1/8 to the right, yielded an $A(z)$ suitable for application to the examples in Figs. 2 and 3. For this example $A(z) = H(ze^{-j\pi/4})$. Six designs for $A(z)$, obtained in the above manner by frequency shifting six

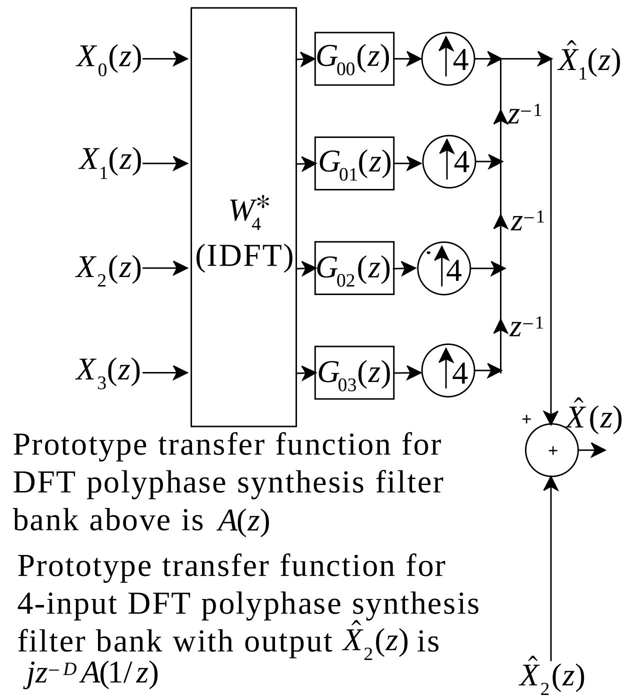
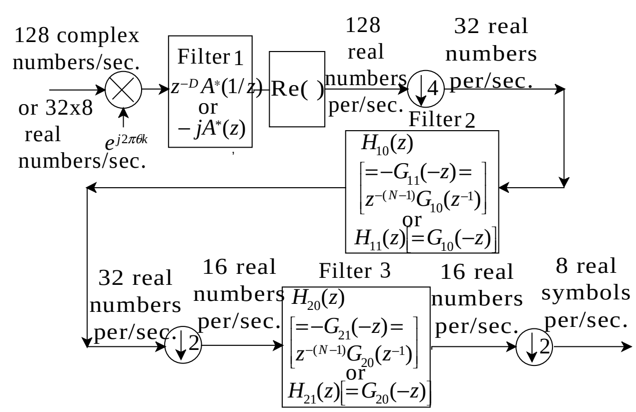


Fig. 2. Arrangement of DFT polyphase synthesis filter banks.



Note 1: The $G_{10}(z)$ and $G_{11}(z)$ are the lowpass and highpass transfer functions of the QMF pair next to the root of the synthesis filter bank trees, and the $G_{20}(z)$ and $G_{21}(z)$ are the transfer functions of the QMF pairs at the leaves of the trees.

Fig. 3. Block diagram of the receiver architecture, and relative symbol rates at various points.

different designs for $H(z)$ with real coefficients, are discussed in [2], and the number of multiplications required for several of these are compared.

Three of the designs considered have linear phase. As noted in [7], and evident from Figs. 2 and 3, the factorization into minimum and maximum phase FIR filters requires that both prototype filters be used at both the transmitter and the receiver. Filters with linear phase are the same in both the

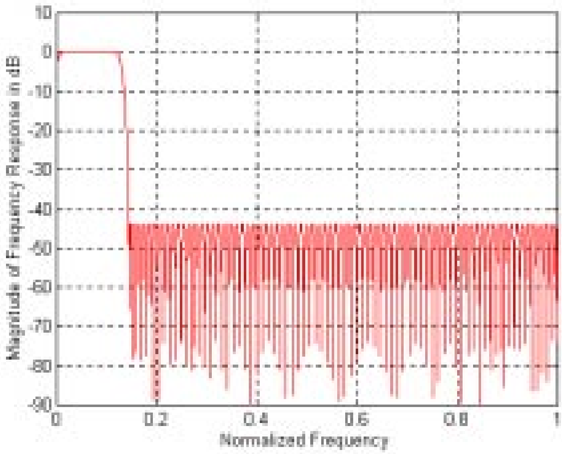


Fig. 4: The magnitude frequency response of the 8th-band equiripple stopband VSB filter used for calculating the polyphase components for the DFT polyphase filter banks in the example.

transmitter and the receiver, and are symmetric, resulting in half the total number of multiplications [2]. On the other hand, the linear phase filters are about 25% longer than those obtained from a factorization into minimum and maximum phase filters. Therefore, by using linear phase designs the overall saving in the total number of multiplications is about 25%.

Lawton [9] has presented another factorization of a Nyquist polynomial, resulting in symmetric filters with complex coefficients. Thus, the number of coefficients would be the same as that obtained by factorization into minimum and maximum phase polynomials, but only one filter type, multiplied, as seen from Figs. 2 and 3, by appropriate constants, would be needed at the transmitter and receiver, so the number of multiplications would be halved. Thus, the computational savings over linear phase designs would be about 25%. Unfortunately, the Lawton factorization cannot provide linear phase [10]. We turn next to factorizations of Nyquist polynomials into polynomials with complex coefficients, and properties of scaling functions and wavelets obtained from the appropriate two-scale difference equations using these complex coefficients [9, 10].

II. FACTORIZATIONS INTO POLYNOMIALS FOR FIR FILTERS WITH COMPLEX COEFFICIENTS AND THE PROPERTIES OF THE CORRESPONDING SCALING FUNCTIONS AND WAVELETS

A. Previous Work

As noted in [10] research on wavelets has been concerned largely with real-valued wavelet bases and perfect reconstruction filter banks (PRFB's). Relatively few publications on complex wavelet bases and filter banks have appeared in the literature: five are cited in [10]. Lawton [9] seems to have been the first to study their properties, and Zhang, *et al.* [10] have deepened and extended Lawton's work significantly. In this paper we apply Lawton's factorization [9] and extensions and ramifications of it by Zhang, *et al.* [10] to the design of FIR filters suitable for design of $A(z)$, following the original ideas in [7]. We also discuss the connections to scaling functions and wavelets, which also have possible applications to communications.

B. Factorizations of Nyquist Filter Polynomials

We summarize the known properties of factorizations of Nyquist filter polynomials into FIR polynomials with complex coefficients and the corresponding scaling functions and wavelet bases [9, 10].

The factorization of the Nyquist filter introduced by Lawton [9] permitted no real roots of $H(z)$ except at $z=-1$, and produced pairs of factors whose zeros were roots and their reciprocals. The resulting magnitude frequency response is the same as that of the filter transfer function with zeros at the roots and their conjugates. Since the zeros occur in reciprocal pairs the filter is symmetric but, as shown in [10], it is not linear phase, as is claimed in [9], even though the scaling function formed by using the coefficients in the proper two-scale difference equation is symmetric and the wavelet is antisymmetric. Based on the theory in [9] McGee [7] proposed using a symmetric filter $H(z)$ to obtain $A(z)$ as a prototype filter in polyphase implementations such as are shown in Figs. 2 and 3. He designed an 87th order Lawton symmetric polynomial with complex coefficients, $P(z)$, which was the transfer function of an 8th band lowpass filter, with a 43-dB stopband loss and a 25% excess bandwidth. It should be noted, however, that this digital filter does not have linear phase. He also derived an architecture for efficiently realizing the filtering at the transmitter and receiver. The use of the Lawton factorization results in a computational saving on the order of 25% over the use of linear phase prototype filters, and 50% over the use of filters obtained from a minimum/maximum phase decomposition. This seems to have been the first application of the filter transfer functions with complex coefficients obtained by using the Lawton factorization to communications or communications signal processing other than source coding applications in image and video processing (references cited in [10]).

Lawton's work resulted in confusion among some readers that symmetry of the scaling function implied linear phase of the corresponding symmetric $H(z)$. This confusion has been cleared up, and connections of the properties of digital filter transfer functions with complex coefficients to the properties of the corresponding scaling functions and wavelets have been provided by Zhang, *et al.* [10]. We summarize these next.

To begin with, Daubechies has shown that all compactly supported real orthogonal wavelet bases and their associated conjugate quadrature filters (CQF's) are neither symmetric nor antisymmetric, except for the Haar function. The first result proven in [10] is that the scaling filter h in a CQF perfect reconstruction filter bank (PRFB) associated with a dyadic compactly supported complex orthogonal wavelet basis cannot be antisymmetric. Hence, it follows that the associated scaling function cannot be antisymmetric either, and more generally, that if complex CQF PRFB's have any symmetry property, then they can only be symmetric, not antisymmetric. It is then shown that for every symmetric CQF for a PRFB with complex coefficients there is a corresponding CQF for a PRFB with real coefficients that has the same magnitude frequency response. The converse does not hold since, for example, it is impossible, for some real Daubechies wavelets, to find corresponding symmetric complex-valued ones.

The next topic dealt with in [10] is the linear phase property. It is shown there that an FIR filter h with complex coefficients that are conjugate symmetric or antisymmetric has linear phase. However, a CQF PFRB filter h that has complex coefficients does not have linear phase if it is symmetric or antisymmetric, contradicting Lawton [9]. A very important result is that the compactly supported dyadic orthogonal complex wavelet bases cannot be linear phase. This is equivalent to the result that a linear-phase CQF with complex coefficients for a PRFB does not exist. It is however, possible to design a CQF with symmetric complex coefficients and to approximate linear phase by using a still different factorization of a Nyquist filter polynomial transfer function, as shown in [10].

We propose the use of an $H(z)$ designed by this method to obtain a suitable $A(z)$ with approximately linear phase that can be used as the prototype filter in polyphase implementations such as those in Figs. 2 and 3. The design should provide computational savings similar to those obtained by using the symmetric $H(z)$ with complex coefficients obtained by using the Lawton factorization [7].

The design of an $H(z)$ with symmetric complex coefficients may begin, for example, by first designing a Nyquist polynomial, $N(z)$, with a desired magnitude frequency response, or by multiplying together two FIR filter transfer functions, $H'(z)$ and $H'(1/z)$, with real coefficients, to obtain $N(z)=H'(z)H'(1/z)$. The $N(z)$ can be factored then into polynomials with complex coefficients as $N(z)=H(z)H^*(z^{-1})$, where the asterisk denotes the conjugation of the coefficients. The complex zeros of $N(z)$ are sets of four, $(z_j, z_j^{-1}, \bar{z}_j, \bar{z}_j^{-1})$. For the Lawton factorization the zeros of $H(z)$ can be selected as z_j and z_j^{-1} (or $\bar{z}_j$ and $\bar{z}_j^{-1}$). We refer the reader to [9] and [10] for a discussion of real roots, which do not often occur in practice. The Lawton factorization yields an $H(z)$ that has complex symmetric coefficients but is not linear phase. To obtain an $H(z)$ that has complex symmetric coefficients but has approximately linear phase it is assumed that all the zeros of $N(z)$ are inside the unit circle, with subscripts as indexes starting with one, ordered so that the arguments of the roots are monotonically increasing within (0, π). Then the odd indexed zeros of $N(z)$ and the reciprocals of the conjugates of the even indexed zeros of $N(z)$ are selected as zeros of $H(z)$, while the even indexed zeros of N(z) and the reciprocals of the conjugates of the odd indexed zeros of $N(z)$ are selected as zeros of $H^*(z^{-1})$. Example designs of both the Lawton type, called normal symmetric complex (NSC), and the approximately linear phase symmetric complex (ALPSC) designs just described, for $N(z)$ constructed from the real coefficients of Daubechies filters are given in [10]. Examples of the symmetric scaling functions and antisymmetric wavelets corresponding to each of the two types of symmetric complex FIR filter designs are also given. Plots of the phase responses of the NSC, ALPSC, normal real (NR), and approximately linear-phase real (ALPR) due to Daubechies demonstrate the superior linearity of an ALPSC design.

The ALPSC design can be used to design a VSB filter $A(z)$ and the other filters related to it, as shown in the examples of Figs. 2 and 3, from $A(z)=H(ze^{-j\pi/4})$ for $M=8$, or for any other M, with $H(z)$ a symmetric FIR transfer function with complex coefficients. Following [7], we next present a summary of the design theory and description of efficient polyphase realizations for $A(z)$ derived from filters $H(z)$ with symmetric complex coefficients.

III. EFFICIENT POLYPHASE REALIZATIONS USING PROTOTYPE VSB FILTERS DERIVED FROM SYMMETRIC FIR FILTERS WITH COMPLEX COEFFICIENTS

From the factorization of the prototype Nyquist filter polynomial $N(z)$ the non-causal filter $H(z)$ is obtained, and then the causal filter is obtained as

$$P(z)=z^{-(L-1)/2}H(z)\,, \qquad (1)$$

where L is the length of the filter [7]. $P(z)$ has coefficients that are complex symmetrical. It may have an arbitrary number of zeros at $z=-1$. The theory of VSB filter banks using such polynomials is as follows. The M transmit filters have M real input sequences each at a rate $2/M$ applied to them. The filter outputs are summed, and the resulting signal is then applied to a set of M receive filters, the real outputs of which are sampled at the rate of $2/M$. The transmit and receive filters are frequency-shifted versions of prototype filters $P(z)$ and $P^*(1/z)$, respectively, with appropriate phasing [6, 7]. The frequency shifts are multiples of $1/M$.

In a well-designed system the real part of the pulse response will be nonzero at the sampling instant, and zero at all times that are displaced from it by multiples of $M/2$, the reciprocal of the sampling rate. The sampling instant occurs $L-1$ sample times after the pulse has been applied at the input. Similarly, in the adjacent channels, the real part of the pulse response should vanish at the main sample time and at sample times displaced from it by multiples of $M/2$.

In the following it is assumed that the length L is a multiple of M: this results in all the polyphase filters having the same length. This assumption may be removed, but this does not appear to lead to great system savings unless low delay is desired.

It is shown in [7] that the transmit filters $T_k(z)$ and receive filters $R_k(z)$ are represented by

$$T_k(z) = e^{j\phi_k} W^{(k+1/2)(L-1)/2} P(W^{k+1/2} z) \qquad (2)$$

$$R_k(z) = e^{-j\phi_k} W^{(k+1/2)(L-1)/2} P^*(W^{k+1/2} z) , \qquad (3)$$

where $W = e^{-j2\pi/M}$ and

$$\phi_{k+1} - \phi_k = \text{an odd multiple of } \pi/2 . \qquad (4)$$

Recalling that the filter length L is a multiple of M and choosing the phase shifts [7] as

$$\phi_k = (k+1/2)(2(L/M)+1)\pi/2 \qquad (5)$$

the transmit and receive filters are represented by

$$T_k(z) = W^{-(k+1/2)(M/2+1)/2} P(W^{k+1/2} z) \qquad (6)$$

$$R_k(z) = W^{(k+1/2)(M/2-1)/2} P^*(W^{k+1/2} z) . \qquad (7)$$

The claimed efficiencies due to computational savings follow because when the phase components of $P(W^{1/2}z)$ are introduced, both the transmitter and receiver involve the multiplication of only one of the sets of prototype filter coefficients for every input set of data. For the DFT the simplest approach seems to be to find the polyphase expansion of $A(z) = W^{-(M/2+1)/4} P(W^{1/2} z)$ as

$$A(z) = \sum_{r=0}^{M-1} z^{-r} A_r(z^M) . \qquad (8)$$

More efficient realizations are obtained using polyphase decompositions. There are two cases, one involving the DFT with elements $e^{j2\pi kr/M}$, and the other involving the odd-time, odd-frequency DFT (OODFT) with elements $e^{j2\pi(k+1/2)(r+1/2)M}$.

Then the output from the transmitter is

$$\sum_{r=0}^{M-1} z^{-r} A_r(z^M) \sum_{k=0}^{M-1} W^{-(r+(M/2+1)/2)k} X_k(z^{M/2}) . \qquad (9)$$

If the integer $M/2+1$ is even then the processing may be accomplished by taking the DFT of the inputs, choosing the $r+(M/2+1)/2$ output of the DFT as the input to the r^{th} polyphase filter, and selecting the output that is desired. Otherwise it is necessary to multiply the M inputs by a phasing factor. This is the OODFT case. Using the DFT leads to the diagram in Fig. 5. For this case the inputs are real sequences and the DFT may be done as a DFT of order $M/2$ on the complex signals $X_{2i} + jX_{2i+1}$. The sampling operation may be moved to just before the delay lines. Also, the output chain of delays may be expressed as a chain of length $M/2$ by incorporating delays $z^{-M/2}$ in the structure. The receiver uses the polyphase expansion of $B(z) = W^{(M/2-1)/2} P^*(W^{1/2} z)$, which is given by

$$B(z) = \sum_{r=0}^{M-1} z^{-r} B_r(z^M) . \qquad (10)$$

Then

$$R_k(z) = \sum_{r=0}^{M-1} z^{-r} B_r(z^M) W^{k(M/2-r-1)} . \qquad (11)$$

A diagram that looks like Fig. 5 reversed gives an efficient realization of the receiver [7]. The arrows on the tapped delay line are at the left and point down instead of up. These are followed by the polyphase components in (11), ordered from top to bottom. Their output signals are the input signals into an M - DFT. The M-DFT is followed by a real part function. The output consists of the real parts of the M-DFT sampled at rate $2/M$. The sampling operation may be moved back to the output of the delay lines. When $M/2$ is an even integer, further phasing is required at the output. For the case when $M/2$ is odd the outputs are real, the DFT may be replaced with a DFT of order M/2.

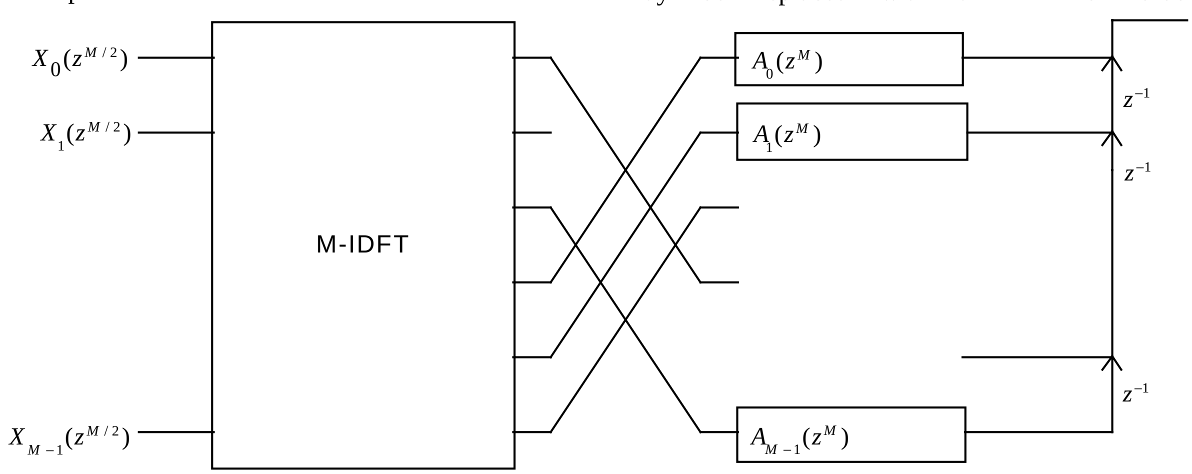


Fig. 5. Transmitter. M real input sequences are applied at rate 2/M to an IFFT transformer, resulting in M complex sequences at rate 2/M. These are then filtered by the M polyphase filters, and the output is obtained by summing the filter outputs properly. If M/2 is even, then a further phasing is necessary at the input.

and appropriate real and imaginary parts of the output taken, and the input delay chain of length M may be replaced with one of length M/2 by incorporating delays of length M/2 within the filtering structure.

The architecture of Fig. 5 may be implemented as shown in Fig. 2 by using the theory and method described in [5] leading up to Fig. 8.21 there, which was used to obtain the architecture in Fig. 2.

IV. CONCLUDING DISCUSSION

We have provided a summary description of a spectrally efficient, bandwidth-on-demand multiplexer-demultiplexer pair in which a computationally efficient DFT polyphase synthesis filter bank was used to implement part of the multiplexer. This polyphase filter bank used a prototype VSB filter $A(z)$ with complex coefficients derived from a lowpass M^{th}-band filter $H(z)$ by frequency translation to the right by $1/M$ using $A(z) = H(ze^{-j\pi/4})$, where M is twice the number of inputs into one of the two IDFT's used in obtaining the output signal of the multiplexer in Fig. 2. In earlier works [2, 4] the $H(z)$ was calculated by factoring a designed Nyquist polynomial in such a way that $H(z)$ had real coefficients. In this paper we describe and discuss two factorizations that result in FIR filters with coefficients that are complex and symmetric. One of these is due to Lawton [9]. The resulting $H(z)$ may be reasonably close to linear phase, but it is not linear phase, even though the associated scaling function is symmetric and the associated wavelet is antisymmetric. Another factorization, due to *Zhang, et al.*, yields approximately linear phase symmetric complex (ALPSC) designs for $H(z)$. These designs for $H(z)$ result in prototype filters $A(z)$ for realization of the polyphase synthesis-analysis pairs in the multiplexer-demultiplexer pairs that are computationally efficient due to the symmetries in the coefficients. The computational efficiencies are the result of savings in the numbers of multiplications of about 25% over linear phase designs, and 50% over designs in which factorizations into minimum and maximum phase polynomials are used. We recommend the ALPSC factorizations yielding filters with symmetric complex coefficients, especially for the recent realizations giving multiplexer channels with linear phase for the tree [11].

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