Innovations-Based Blind Sequence Detection for Wireless Channels

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Abstract— **A blind sequence detection algorithm based on the innovations approach is proposed and its performance in a slow and flat Rayleigh fading environment is evaluated. A comparison between the innovations and the Singular Value Decomposition (SVD) [1] based blind sequence detection algorithm is also presented. Simulation results show that the innovations blind sequence detector has about a 3 dB gain over the SVD blind sequence detector in the fading model studied in this paper. An analysis for various performance measures will be presented to explain the behaviour of these blind sequence detectors in the fading environment.**

I. Introduction

In order to have high-speed reliable communication on a wireless channel, channel identification and equalization are required to combat ISI (Intersymbol Interference). Normally, channel identification and equalization are done either by sending long training sequences or by designing the equalizer based upon prior knowledge of the channel. Unfortunately, in radio communications little is known about the channel a priori and many standard adaptive detectors used in radio environments waste some of their transmission time on a training sequence. Many blind equalization algorithms have been proposed and developed and as a result have made this area of research very important in the recovery of signals corrupted by unknown channel disturbances.

While a lot of work has been done in developing blind equalization techniques, not much research has been done in the area of blind sequence detection. In blind sequence detection, the training period of the transmission over an ISI channel can be reduced or eliminated and the convergence rate occurs within 100 symbols. Many mobile channels require channel identification within 100 symbols [1].

One approach to blind sequence detection, based on Singular Value Decomposition (SVD), has been proposed in [1]. It can be shown that a relationship exists [3] between the innovations process [5] and SVD. Therefore it is theoretically possible to derive an innovations based blind sequence detector (I-BSD). The innovations process has found numerous applications in the area of wireless communications. Yu and Pasupathy [2] have applied the innovations approach to Rayleigh fading channels, and have developed a general and practical MLSE (Maximum Likelihood Sequence Estimation) receiver which demodulates a received signal recursively in a non-coherent fashion. So far in the current literature, nobody has studied the application of an innovations approach to blind sequence detection (especially in the case of fading channels) or compared it to the SVD based blind sequence detection algorithm proposed in [1].

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In this paper we aim to achieve two objectives. The first objective is to present an alternative method to blind sequence detection. We propose a blind sequence detection algorithm based on the innovations approach. One of the main motivations of using an innovations approach for blind sequence detection is that it has many inherent advantages over SVD. The innovations approach is simple, causal and computationally efficient system when compared to SVD [3] [4]. Our approach does not require any channel identification for detection. The fundamental idea behind our methodology is to estimate the source (deterministic) correlation without knowing the channel characteristics, and then to use the Viterbi algorithm to obtain the source symbols. We will show that in order to estimate the signal correlation function, only the orthogonalization of the channel is necessary. The innovations approach is used as a motivation for this orthogonalization. Our second objective is to compare the proposed I-BSD and the SVD based blind sequence detector (SVD-BSD). The operation of these two detectors in a slow and flat Rayleigh fading channel will be analysed.

II. System Model and Assumptions

In this section we will briefly outline the system model and parameters used to develop our innovations based blind sequence detector. This model is similar to the one used in [1]. We can write a vector form of the received version of a signal transmitted in a multipath Rayleigh fading channel as

$$\mathbf{x}(t) = \mathbf{H}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{s}(t)$ represents the symbol sequence where only one symbol is transmitted for every time interval T_s, $\mathbf{H}$ represents the combined channel matrix consisting of the fading parameters, pulse shaping and receiver filters, and $\mathbf{n}(t)$ is the AWGN vector. The main idea behind blind sequence detection is to detect the information symbol s_n without knowing what the channel parameter $\mathbf{H}$ is. The parameter d represents the dimension of the signal subspace. The following assumptions will be made in our model [1]:

(A1) The information symbol sequence s_n is zero mean and $E(s_i s_j^*) = \delta(i-j)$.

(A2) Noise power $n_j()$ is zero mean for all j and $E(n_i(t_1)n_j^*(t_2)) = \sigma^2\delta(i-j)\delta(t_1-t_2)$.
(A3) The noise process is uncorrelated with $\{s_n\}$.
(A4) Channel identification matrix $\mathbf{H}$ is an N by d matrix with full column rank.

III. An Innovations Based Blind Sequence Detector

A. Innovations and Channel Orthogonalization

The innovations process plays an important role in our blind sequence detector. We will show that, in order to estimate the signal correlation function, one only has to orthogonalize the channel using only the observation data. The innovations approach is used as a motivation for this orthogonalization. Let

$$\mathbf{R}_x(k) = E(\mathbf{x}(t)\mathbf{x}^*(t-k)). \quad (2)$$

From (2) and assumptions A1 to A4 we get

$$\mathbf{R}_x(0) = \mathbf{H}\mathbf{H}^* + \sigma^2\mathbf{I}. \quad (3)$$

By the innovations approach or Cholesky factorization [5] $\mathbf{R}_x(0)$ can be written as

$$\mathbf{R}_x(0) = \mathbf{P}^{-1}\mathbf{D}(\mathbf{P}^*)^{-1} \quad (4)$$

where $\mathbf{P}$ is a lower triangular matrix composed of all orders of prediction coefficients of $\mathbf{x}(t)$ and $\mathbf{D}$ is a diagonal matrix where the diagonal entries are the variances of the corresponding prediction errors. If there is no noise in (3) ($\sigma^2 = 0$) we then have

$$\begin{aligned} \mathbf{H}\mathbf{H}^* &= \mathbf{P}^{-1}\mathbf{D}(\mathbf{P}^*)^{-1} \quad (5)\\ (\mathbf{D}^{-1/2}\mathbf{P}\mathbf{H})(\mathbf{D}^{-1/2}\mathbf{P}\mathbf{H})^* &= \mathbf{I}. \quad (6) \end{aligned}$$

Therefore, the transform $\mathbf{\Gamma} = \mathbf{D}^{-1/2}\mathbf{P}$ orthogonalizes the channel $\mathbf{H}$. In otherwords, there is an orthogonal matrix $\mathbf{V}$ such that $\mathbf{\Gamma}\mathbf{H} = \mathbf{V}$. The matrix $\mathbf{\Gamma}$ is called the whitening filter and its inverse $\mathbf{L} = \mathbf{\Gamma^{-1}}$ is called the innovations filter of $\mathbf{x}(t)$. In the absence of noise, the (deterministic) correlation of the Γ-transformed output

$$\mathbf{y}(t) = \mathbf{\Gamma}\mathbf{x}(t) = \mathbf{V}\mathbf{s}(t) \quad (7)$$

is identical to that of the source [1]

$$\mathbf{y}^*(t)\mathbf{y}(t-k) = \mathbf{s}^*(t)\mathbf{s}(t-k). \quad (8)$$

Without having any knowledge of the channel, the (deterministic) correlation of $\mathbf{s}(t)$ can be found from the received samples.

B. Implementation of the Viterbi Algorithm

The key point of the blind sequence detection algorithm is that the inner product (8) is preserved. The Viterbi Algorithm can be used for a noisy channel to calculate the estimated sequence which results in the minimum correlation (deterministic) difference between the Γ-transformed received sample $y(t)$ and the estimated sequences. If we let

$$\begin{aligned} r_y^{(k)}(t) &= \mathbf{y}^*(t)\mathbf{y}(t-k) \quad (9)\\ r_s^{(k)}(t) &= \mathbf{s}^*(t)\mathbf{s}(t-k) \quad (10)\\ &= \sum_{l=0}^{d-1} s_{t-l}^* s_{t-l-k} \quad (11) \end{aligned}$$

then one obtains

$$r_y^{(k)}(t) = r_s^{(k)}(t) + w^{(k)}(t) \quad (12)$$

where $w^{(k)}(t)$ represents the interfering noise components of the observation. We can use the definition found in [1] to state the optimum sequence detection as:

$$min \sum_t |r_y^{(k)}(t) - r_s^{(k)}(t)|^2. \quad (13)$$

This detection problem can be solved by applying the Viterbi Algorithm to a K^{d+k-1} state trellis for a K-QAM constellation. The delay parameter k should be chosen in such a manner so that the number of states in the Viterbi Algorithm is minimized. Increasing k results in a more complex system and as shown in [3], the added complexities of a higher k are not very beneficial to blind sequence detection algorithms. Thus we set $k = 1$ as being sufficient to determine the information sequence s_n.

C. Source Correlation Estimators

The innovations process only allows us to find the source correlation of $\mathbf{s}(t)$ from the correlation of $\mathbf{y}(t)$ in the absence of noise. In many communication systems, noise is a common phenomena and must be dealt with. Unfortunately, the whitening filter $\mathbf{\Gamma}$ is not optimum in the presence of noise. The main goal of the I-BSD is to obtain an optimum estimation of the source correlation

$$r_s(t) = \mathbf{s}^*(t)\mathbf{s}(t-1). \quad (14)$$

The Viterbi Algorithm can then be used with the estimated $r_s(t)$ to recover the input symbols. We can use the innovations process as a motivation to estimate $r_s(t)$ from the correlation function of the transformed observations. Define

$$\mathbf{y}(t) = \mathbf{\Gamma}_o\mathbf{x}(t) \quad (15)$$

$$r_y(t) = \mathbf{y}^*(t)\mathbf{y}(t-1). \quad (16)$$

The goal is to find an optimum filter $\mathbf{\Gamma}_o$ that minimizes [1]

$$J(\mathbf{\Gamma}) = E(|r_y^{(k)}(t) - r_s^{(k)}(t)|^2). \quad (17)$$

Solving for the optimum filter $\mathbf{\Gamma_o}$ for the matrix equation that arises by minimizing (16) does not seem straight forward and we were unable to find a closed form solution. It turns out that the minimum variance estimate of the source correlation for a one dimensional system ($d = 1$)

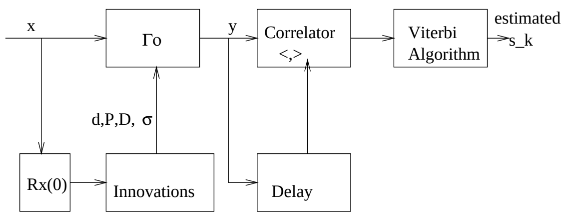

Fig. 1. An innovations based blind sequence detection algorithm.

and the linear transform matrix $\mathbf{\Gamma}_{mv}$ that gives the minimum variance estimate [6] of $\mathbf{s}(t)$ have the same form which is:

$$\mathbf{\Gamma}_o = (\mathbf{D} - \sigma^2 \mathbf{P}\mathbf{P}^*)^{1/2} \mathbf{D}^{-1} \mathbf{P}. \qquad (18)$$

Intuitively it seems that the filter in (18) may be well suited for an I-BSD. A schematic of the innovations approach to blind sequence detection is shown in Figure 1.

IV. Simulation Results: An Analysis of Blind Sequence Detectors

In this section we present a new fading model different than the one used in [1]. Only two ray multi-path channels were used where we assumed to have only one receiver. The composite channel ($h(t)$) used in our model can be described as

$$h(t) = \alpha_1 p(t) \; + \; \alpha_2 p(t - \tau) \qquad (19)$$

where $p()$ was a raised cosine pulse with 90% roll off. The time delays τ were delayed from $\tau = 0.1T$ to $\tau = 1.0T$. The time delays were purposely varied in order to see how the blind sequence detectors behaved in different "delay spread" environments. This particular model was chosen because we wish to see how different types of blind sequence detectors work in an environment where we can easily vary channel parameters and see their direct effect on the performance of blind sequence detection algorithms. By keeping the model relatively simple yet realistic, we can gain more insight on how blind sequence detection algorithms operate. The gain (α_i) was generated from a zero mean unit variance Gaussian distribution.

In Figure 2 the optimal results in terms of BER for the fading model using the SVD-BSD and I-BSD have been displayed. Notice that for both blind sequence detectors, as the delay spread τ increases so does the SNR for a fixed BER. Therefore it appears that both blind sequence detection algorithms are sensitive to changes in the delay spread. The SVD-BSD is especially sensitive when τ goes from 0.4T to 0.5T.

We introduce a new measure known as the average correlation estimation error (CER) to explain why a certain detector works well in a particular environment. This measure is defined as:

$$CER = \frac{\sum_{i=1}^{N} |\mathbf{r}_y(i) - \mathbf{r}_s(i)|^2}{N}. \qquad (20)$$

Here, $\mathbf{r}_y(i)$ is the ith element of the vector $\mathbf{r}_y$. Recall that $\mathbf{r}_y$ is a vector of the estimated source correlation

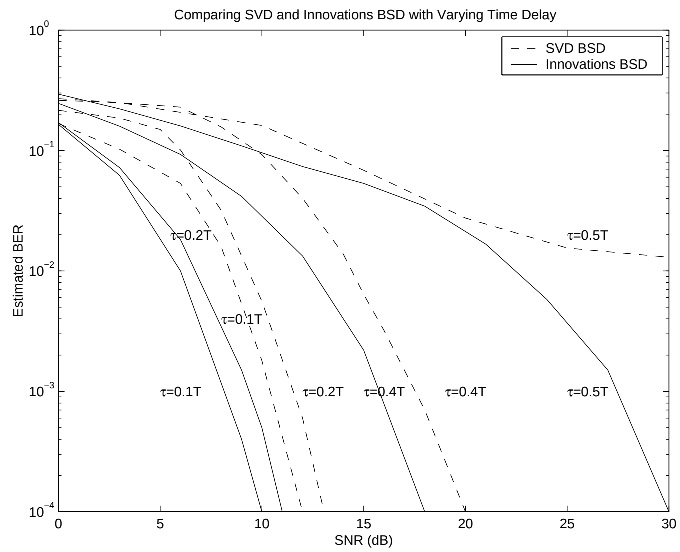

Fig. 2. Varying the delay spread τ for the Innovations and SVD blind sequence detectors.

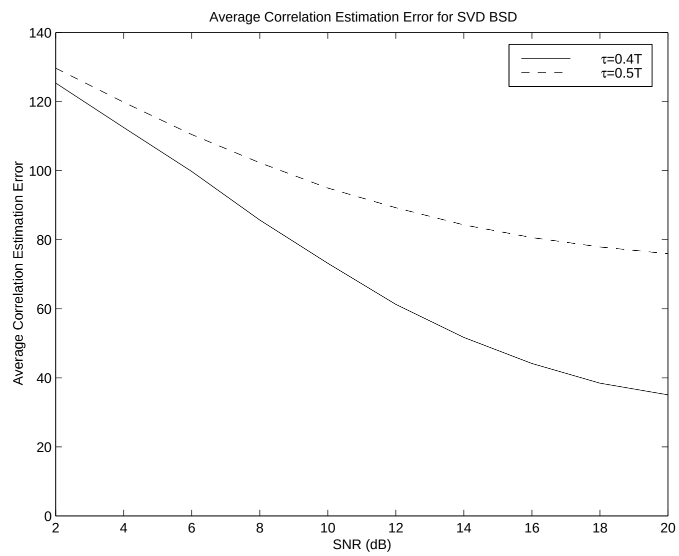

Fig. 3. Average correlation estimation error for SVD blind sequence detectors with varying time delay.

determined by the blind sequence detector. Similarly, $\mathbf{r}_s$ is a vector of the actual source correlation where $\mathbf{r}_s(i)$ is the ith element in $\mathbf{r}_s$. The parameter N is the number of correlation entries in the vector. If we analyze the CER for the SVD BSD where τ is varied from 0.4T to 0.5T, we see that the CER increases for a particular SNR as we increase τ (see Figure 3). This increase in the estimation error results in a BER increase since at a larger delay spread the SVD BSD poorly estimates the source correlation.

In Figure 2, we see that by using the I-BSD, there is a significant gain of about 3 dB when compared to the SVD-BSD. By looking at the CER measure more insight is gained as to why the I-BSD has a better performance. However, in order to make an accurate comparison when

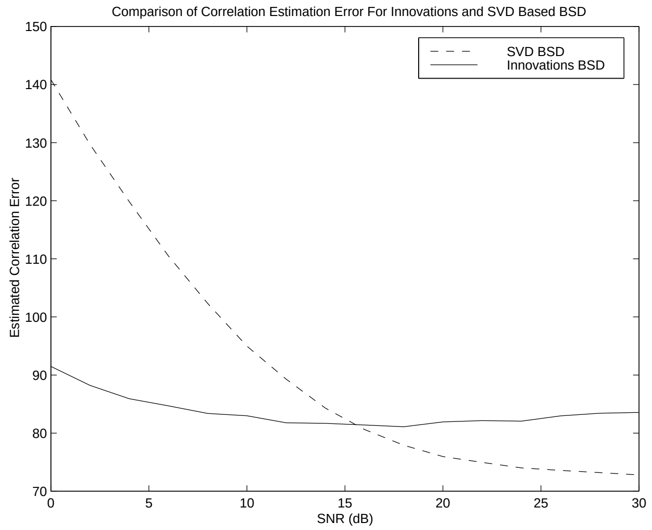

Fig. 4. Correlation estimation error for innovations BSD ($d = 1$) and SVD BSD ($d = 2$) with $\tau = 0.5T$.

using the CER measure, the same d (signal subspace dimension) should be used for both blind sequence detectors. Unfortunately, the optimal d for the I-BSD is different than the SVD-BSD. In Figure 4, the correlation estimation error is plotted for both estimators where $d = 1$ for the I-BSD and $d = 2$ for the SVD-BSD. Initially, the I-BSD has a lower estimation error. However, when the SNR goes above 16 dB the SVD-BSD has a lower estimation error yet in terms of the BER the SVD-BSD still has a much higher BER than the I-BSD.

As mentioned earlier, we are using different signal dimensions (d) for these estimators and as such a fair comparison can not be made since both estimators are using a different trellis for their Viterbi Algorithm. However, we can still give some explanation as to why the I-BSD has a better performance. In Figures 5 and 6, the trellises for $d = 1$ and $d = 2$ with the minimum error event path is shown. If an error event does happen when $d = 2$, it results in more bit errors than in the case for $d = 1$. Notice that when $d = 1$, only 1 bit is in error for the minimum error path while for $d = 2$ two bits are in error. Since the SVD-BSD has a higher d parameter, when an error event does occur more bit errors will result. Eventhough the SVD-BSD has a slightly lower correlation estimation error, it is susceptible to more bit errors because it has a higher d parameter than the I-BSD. We can conclude that one must be careful when looking at estimation error as an absolute performance measure. A low estimation error does not necessarily result in a lower BER especially in the case when different trellises are used for the Viterbi Algorithm.

V. Conclusions

An innovations approach to blind sequence detection was studied and compared to the SVD-BSD. Each of these techniques has its advantages and disadvantages.

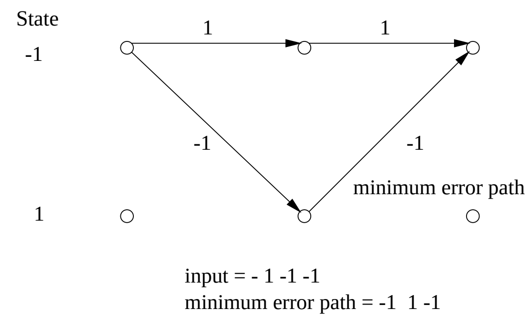

Fig. 5. Trellis with $d = 1$ with a minimum error event path.

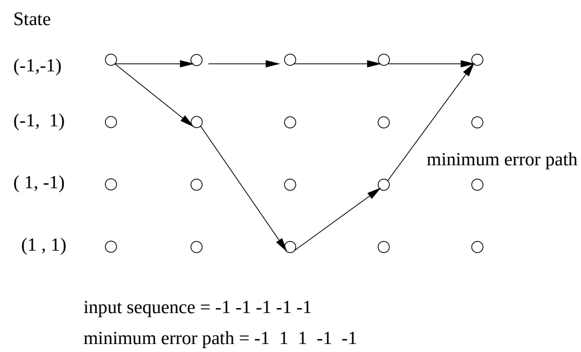

Fig. 6. Trellis with $d = 2$ with a minimum error event path.

The obvious benefits of using the innovations approach are that it is a causal, simpler, less expensive and computationally more efficient system than SVD. An innovations based blind sequence detector for Rayleigh fading channels was proposed and its performance was studied using computer simulations. Simulations were also carried out to compare the error performance of both the SVD and innovations based blind sequence detection algorithms. When the two estimators were compared in a slow and flat fading channel, the I-BSD outperformed the SVD-BSD. By looking at the correlation estimation error measure and trellis diagrams for both blind sequence detectors insight was gained as to explain why a certain estimator performed better than the other for a particular environment.

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