Effect of Imperfect Channel Estimation on the Performance of Multiuser Detectors over Multipath Rayleigh Fading Channel

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Abstract— **In this paper, the performances of two classes of decorrelating detectors, the RAKE decorrelating detector (RDD) and the multipath decorrelating detector (MDD), in a synchronous DS/CDMA system in frequency-selective Rayleigh fading channel are investigated when there exists some mismatch between the channel estimates and the true values. The probability of error and the near-far resistance of these two detectors are examined in the presence of three classes of channel estimation mismatch, i.e., the mismatch of timing, phase estimation and amplitude estimation of the received complex-valued signals. Numerical results show that, while the RDD has a lower probability of error than the MDD on the basis of perfect channel estimation, it is more sensitive to the channel estimation mismatch.**

I. Introduction

One of the active research areas in the last few years is multiuser detection, whose primary objective is to demodulate mutually interfering digital streams of information reliably with minimal computational complexity. Among all the schemes proposed, the linear decorrelating detector (DD) is appealing for its near-far resistance [1] [2], which is especially valuable in the wireless DS/CDMA communication environment.

In the process of demodulation, the DD utilizes the knowledge of channel parameters, including time delays and received signal phase. In practice, all these parameters must be estimated by the receiver and are subject to estimation errors. The influence of mismatched channel estimation on the DD in single-path Rayleigh fading channel was presented in [3]. However, realistic mobile radio channel is more suitable to be characterized by multipath fading. In [4] and [5], two classes of linear DDs in multipath fading channel were proposed: one is the RAKE decorrelating detector (RDD), which uses a bank of RAKE matched filters (MF) at the front end followed by a linear transformation, and the other is the multipath decorrelating detector (MDD), in which the signals of different paths from the same user in a multipath fading channel are assumed as signals from different users in a single-path channel. The demodulation of the MDD begins with passing the received signal through a bank of filters matched to the assumed single-path user signals. The decision is made by taking the sign of the maximal ratio combined output of the linear transformation of the MDD.

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In this paper, the relations between the error probability and the near-far resistance of these two DDs and the quantity of the channel estimation mismatch are studied. The analysis and the numerical results show that the error probability and the near-far resistance of the RDD degrades at a higher rate than those of the MDD as the amount of the channel estimation mismatch increases.

II. System Model and Performance Analysis

We consider a BPSK synchronous DS/CDMA transmission in a frequency-selective multipath Rayleigh fading channel. It is assumed that the fading channel is slowly variant compared to the transmission rate and that the maximal multipath propagation delay is much shorter than the symbol interval. The first assumption implies that the channel is relatively constant during one symbol interval and the second one implies that the effect of intersymbol interference (ISI) between the transmitted information symbols can be neglected. Under the assumptions made, the channel response for the kth user signal can be given as [6]

$$c_k(t) = \sum_{l=1}^{L_k} c_{kl}\delta(t-\tau_{kl}) = \sum_{l=1}^{L_k} |c_{kl}|e^{j\alpha_{kl}}\delta(t-\tau_{kl}) \qquad (1)$$

where L_k is the number of resolvable paths of the received signal for the kth user, τ_{kl} and c_{kl} are the propagation delay and the channel coefficient of the lth path for the kth user, respectively. In a Rayleigh fading channel, the channel coefficient c_{kl} is a zero mean, complex Gaussian random variable, whose amplitude $|c_{kl}|$ and phase α_{kl} satisfy Rayleigh distribution and uniform distribution from 0 to 2π, respectively. At the receiver, the received signal can be expressed as

$$y(t) = \sum_{k=1}^{K} A_k b_k \sum_{l=1}^{L_k} c_{kl} s_k(t-\tau_{kl}) + \sigma n(t) \qquad (2)$$

where K is the number of the active users, A_k and b_k are the signal amplitude and the information bit transmitted by the kth user, respectively. $s_k(t)$ is the kth user's signature waveform which only takes the nonzero value in the interval $[0,\ T_b]$ and is normalized to have unit energy, i.e., $\|s_k(t)\|^2 = \int_0^{T_b} s_k^2(t)dt = 1$. $n(t)$ is the complex additive white Gaussian noise (AWGN) with unit power spectral density. For simplicity, in what follows, $L_1 = L_2 = \cdots = L_K = L$ is assumed.

A. RAKE Decorrelating Detector (RDD)

In the RDD, the virtual signature waveform for the kth user is assumed to be the superposition of L replicas of $s_k(t)$ from L different paths, which is given by

$$h_k(t) = \sum_{l=1}^{L} |c_{kl}| \, e^{j\alpha_{kl}} s_k(t-\tau_{kl}) \tag{3}$$

With perfect channel estimation, the outputs of the MF bank are denoted as

$$y_k = \int_{\tau_{k0}}^{T_b+\tau_{k0}} (h_k(t))^* y(t) dt \qquad \text{for } 1 \le k \le K \tag{4}$$

In matrix form, (4) can be written as

$$\mathbf{y} = \mathbf{RAb} + \mathbf{n} \tag{5}$$

where $\mathbf{y} = [y_1 \; y_2 \; \cdots \; y_K]^T$, $\mathbf{A} = \text{diag}\{A_1, \; A_2, \; \ldots, \; A_K\}$, $\mathbf{b} = [b_1 \; b_2 \; \cdots \; b_K]^T$, $\mathbf{R} = \{\rho_{ij}\}$ with $\rho_{ij} = \int_{\tau_{i0}}^{T_b+\tau_{i0}} (h_i(t))^* h_j(t) dt$. $\mathbf{n}$ is a complex-valued Gaussian random vector with independent real and imaginary components, whose covariance matrix is $2\sigma^2\mathbf{R}$. Without loss of generality, the detection of b_1 will be considered in the following analysis. In the RDD, the decision for the information bit b_1 is given as

$$\hat{b}_1 = \text{sgn}(\text{Re}\{(\mathbf{R}^{-1}\mathbf{y})_1\}) = \text{sgn}(A_1 b_1 + \tilde{n}_1) \tag{6}$$

the variance of $\tilde{n}_1$ is equal to $\sigma^2(\mathbf{R}^{-1})_{11}$ [7].

In practice, however, the channel estimation may not be perfect. Hence, some performance degradation may appear when the demodulation is based on the imperfect channel estimates. Denoting the estimation values of $|c_{kl}|$, α_{kl} and τ_{kl} by $|\hat{c}_{kl}|$, $\hat{\alpha}_{kl}$ and $\hat{\tau}_{kl}$, the estimated virtual signature waveform for the kth user is given by

$$\hat{h}_k(t) = \sum_{l=1}^{L} |\hat{c}_{kl}| \, e^{j\hat{\alpha}_{kl}} s_k(t-\hat{\tau}_{kl}) \qquad \text{for } 1 \le k \le K \tag{7}$$

Then, the outputs of the MF bank are denoted by

$$\hat{y}_k = \int_{\hat{\tau}_{k0}}^{T_b+\hat{\tau}_{k0}} (\hat{h}_k(t))^* y(t) dt \qquad \text{for } 1 \le k \le K. \tag{8}$$

In matrix form,

$$\hat{\mathbf{y}} = \hat{\mathbf{R}}\mathbf{Ab} + \mathbf{n}_c \tag{9}$$

where $\hat{\mathbf{y}} = [\hat{y}_1 \; \hat{y}_2 \; \cdots \; \hat{y}_K]^T$, $\hat{\mathbf{R}} = \{\hat{\rho}_{ij}\}$ with $\hat{\rho}_{ij} = \int_{\hat{\tau}_{i0}}^{T_b+\hat{\tau}_{i0}} (\hat{h}_i(t))^* h_j(t) dt$. $\mathbf{n}_c$ is the noise term with the covariance matrix equal to $2\sigma^2\hat{\mathbf{R}}_c$, where $\hat{\mathbf{R}}_c = \{\hat{\rho}_{cij}\}$ with $\hat{\rho}_{cij} = \int_{\hat{\tau}_{i0}}^{T_b+\hat{\tau}_{i0}} (\hat{h}_i(t))^* \hat{h}_j(t) dt$.

Based on the channel estimates, the decision for b_1 is made as

$$\begin{aligned} \hat{b}_1 &= \text{sgn}(\text{Re}\{(\hat{\mathbf{R}}_c^{-1}\hat{\mathbf{y}})_1\}) \\ &= \text{sgn}(\sum_{k=1}^{K} A_k \{\text{Re}[\hat{\mathbf{R}}_c^{-1}\hat{\mathbf{R}}]\}_{1,k} b_k + \tilde{n}_{c1}) \end{aligned} \tag{10}$$

the variance of $\tilde{n}_{c1}$ is $\sigma^2(\hat{\mathbf{R}}_c^{-1})_{11}$. By defining

$$B_k = A_k\{\text{Re}[\hat{\mathbf{R}}_c^{-1}\hat{\mathbf{R}}]\}_{1,k} \qquad \text{for } 1 \le k \le K \tag{11}$$

the error probability of the RDD is given by [8]

$$P_1 = \frac{1}{2^{K-1}} \sum_{b_2 \cdots b_K \in \{-1,+1\}^{K-1}} Q\left[\frac{B_1 + \sum_{k=2}^{K} B_k b_k}{\sigma\sqrt{(\hat{\mathbf{R}}_c^{-1})_{11}}}\right] \tag{12}$$

B. Multipath Decorrelating Detector (MDD)

In the MDD, a filter bank is applied in which filters are matched to the replicated signals from different paths of each user. With perfect channel estimation, the outputs of the filter bank are

$$y_{kl} = \int_{\tau_{kl}}^{T_b+\tau_{kl}} s_k(t-\tau_{kl}) y(t) dt \qquad \text{for } 1 \le k \le K; 1 \le l \le L \tag{13}$$

In matrix form, (13) is expressed as

$$\mathbf{y} = \mathbf{MC\Lambda B} + \mathbf{n} \tag{14}$$

where

$$\begin{aligned} \mathbf{y} &= [y_{11} \; \cdots \; y_{1L} \; y_{21} \; \cdots \; y_{2L} \; \cdots \; y_{K1} \; \cdots \; y_{KL}]^T \\ \mathbf{C} &= \text{diag}\{\mathbf{C}_1, \; \mathbf{C}_2, \; \ldots, \; \mathbf{C}_K\} \\ \mathbf{C}_k &= \text{diag}\{e^{j\alpha_{k1}}, \; e^{j\alpha_{k2}}, \; \ldots, \; e^{j\alpha_{kL}}\} \qquad \text{for } 1 \le k \le K \\ \mathbf{\Lambda} &= \text{diag}\{\mathbf{\Lambda}_1, \; \mathbf{\Lambda}_2, \; \ldots, \; \mathbf{\Lambda}_K\} \\ \mathbf{\Lambda}_k &= \text{diag}\{A_k |c_{k1}|, \; A_k |c_{k2}|, \; \ldots, \; A_k |c_{kL}|\} \\ & \qquad \text{for } 1 \le k \le K \\ \mathbf{B} &= [\mathbf{b}_1^T \; \mathbf{b}_2^T \; \cdots \; \mathbf{b}_K^T]^T \end{aligned}$$

$\mathbf{b}_k$ is a column vector whose L entries are all equal to b_k. $\mathbf{M}$ is the cross correlation matrix which can be represented by the set of matrix blocks $\{\mathbf{M}_{ij}\}$ ($1 \le i, \; j \le K$), and the mnth component of the matrix $\mathbf{M}_{ij}$ is $M_{ij}(m,n) = \int_{\tau_{im}}^{T_b+\tau_{im}} s_i(t-\tau_{im}) s_j(t-\tau_{jn}) dt$ ($1 \le m, \; n \le L$). $\mathbf{n}$ is the noise term with the covariance matrix $2\sigma^2\mathbf{M}$. Similarly, the demodulation of b_1 is of interest here. The decision for b_1 is determined by the combination of the L entries of the vector given by

$$\mathbf{z}_1 = \text{Re}\{\mathbf{C}_1^H(\mathbf{M}^{-1}\mathbf{y})_{1:L}\} = \mathbf{\Lambda}_1\mathbf{b}_1 + \tilde{\mathbf{n}}_1 \tag{15}$$

where $\tilde{\mathbf{n}}_1$ has covariance matrix equal to $\sigma^2(\mathbf{M}^{-1})_{1:L,1:L}$.

Before the combination, the colored noise $\tilde{\mathbf{n}}_1$ is whitened by applying the Cholesky factorization in order to reduce the noise energy. Denoting $\mathbf{F}_1$ as the matrix obtained by Cholesky factorization of $(\mathbf{M}^{-1})_{1:L,1:L}$, i.e., $(\mathbf{M}^{-1})_{1:L,1:L} = \mathbf{F}_1^T\mathbf{F}_1$, then

$$\tilde{\mathbf{z}}_1 = \mathbf{F}_1^{-T}\mathbf{z}_1 = \mathbf{F}_1^{-T}\mathbf{\Lambda}_1\mathbf{b}_1 + \tilde{\tilde{\mathbf{n}}}_1 \tag{16}$$

the covariance matrix of $\tilde{\tilde{\mathbf{n}}}_1$ is $\sigma^2\mathbf{I}$.

Now consider the detection based on the estimated channel coefficients. Denoting $\hat{\tau}_{kl}$ as the estimate of τ_{kl}, the outputs of the MF bank are given by

$$\hat{y}_{kl} = \int_{\hat{\tau}_{kl}}^{T_b+\hat{\tau}_{kl}} s_k(t-\hat{\tau}_{kl})y(t)dt \quad \text{for } 1 \le k \le K; 1 \le l \le L \tag{17}$$

In matrix form,

$$\hat{\mathbf{y}} = \hat{\mathbf{M}}\mathbf{C}\mathbf{\Lambda}\mathbf{b} + \mathbf{n}_c \tag{18}$$

where $\hat{\mathbf{y}} = [\hat{y}_{11} \; \cdots \; \hat{y}_{1L} \; \hat{y}_{21} \; \cdots \; \hat{y}_{2L} \; \cdots \; \hat{y}_{K1} \; \cdots \; \hat{y}_{KL}]^T$, $\hat{\mathbf{M}} = \{\hat{\mathbf{M}}_{ij}\}$ $(1 \le i, \; j \le K)$, and the mnth component of the matrix $\hat{\mathbf{M}}_{ij}$ is $\hat{M}_{ij}(m,n) = \int_{\hat{\tau}_{im}}^{T_b+\hat{\tau}_{im}} s_i(t-\hat{\tau}_{im})s_j(t-\tau_{jn})dt$ $(1 \le m, \; n \le L)$. $\mathbf{n}_c$ is the Gaussian noise which is characterized by $E[\mathbf{n}_c\mathbf{n}_c^H] = 2\sigma^2\hat{\mathbf{M}}_c$, where $\hat{\mathbf{M}}_c = \{\hat{\mathbf{M}}_{cij}\}$ $(1 \le i, \; j \le K)$ and the mnth component of $\hat{\mathbf{M}}_{cij}$ is $\hat{M}_{cij}(m,n) = \int_{\hat{\tau}_{im}}^{T+\hat{\tau}_{im}} s_i(t-\hat{\tau}_{im})s_j(t-\hat{\tau}_{jn})dt$ $(1 \le m, \; n \le L)$.

On the basis of the channel estimates, the decision of b_1 is made by combining the L entries of the vector

$$\begin{aligned} \hat{\mathbf{z}}_1 &= \mathrm{Re}\{\hat{\mathbf{C}}_1^H(\hat{\mathbf{M}}_c^{-1}\hat{\mathbf{y}})_{1:L}\} \\ &= \mathrm{Re}\{\hat{\mathbf{C}}_1^H(\hat{\mathbf{M}}_c^{-1}\hat{\mathbf{M}}\mathbf{C}\mathbf{\Lambda}\mathbf{B})_{1:L}\} + \tilde{\mathbf{n}}_{c1} \\ &= \mathbf{r}_1 + \tilde{\mathbf{n}}_{c1} \end{aligned} \tag{19}$$

where $\hat{\mathbf{C}}_1 = \mathrm{diag}\{e^{j\hat{\alpha}_{11}}, \; e^{j\hat{\alpha}_{12}}, \; \ldots, \; e^{j\hat{\alpha}_{1L}}\}$ and $\hat{\alpha}_{kl}$ is the estimate of α_{kl}. The covariance matrix of $\tilde{\mathbf{n}}_{c1}$ is $\sigma^2(\hat{\mathbf{M}}_c^{-1})_{1:L,1:L}$. To whiten the colored Gaussian noise, $\hat{\mathbf{z}}_1$ is multiplied by the matrix $\hat{\mathbf{F}}_1$,

$$\hat{\mathbf{F}}_1^{-T}\hat{\mathbf{z}}_1 = \hat{\mathbf{F}}_1^{-T}\mathbf{r}_1 + \tilde{\tilde{\mathbf{n}}}_{c1} \tag{20}$$

where $\hat{\mathbf{F}}_1$ is the matrix obtained by the Cholesky factorization of $(\hat{\mathbf{M}}_c^{-1})_{1:L,1:L}$, i.e., $(\hat{\mathbf{M}}_c^{-1})_{1:L,1:L} = \hat{\mathbf{F}}_1^T\hat{\mathbf{F}}_1$. And $\tilde{\tilde{\mathbf{n}}}_{c1}$ is characterized by $E[\tilde{\tilde{\mathbf{n}}}_{c1}\tilde{\tilde{\mathbf{n}}}_{c1}^T] = \sigma^2\mathbf{I}$.

After the maximal ratio combining, the probability of error is given by

$$\begin{aligned} P_1 &= \frac{1}{2^K} \sum_{b_1 \cdots b_K \in \{-1,+1\}^K} Q\left[\sqrt{\frac{(\hat{\mathbf{F}}_1^{-T}\mathbf{r}_1)^T\hat{\mathbf{F}}_1^{-T}\mathbf{r}_1}{\sigma^2}}\right] \\ &= \frac{1}{2^K} \sum_{b_1 \cdots b_K \in \{-1,+1\}^K} Q\left[\sqrt{\frac{\mathbf{r}_1^T((\hat{\mathbf{M}}_c^{-1})_{1:L,1:L})^{-1}\mathbf{r}_1}{\sigma^2}}\right] \end{aligned} \tag{21}$$

III. Numerial Results

Let us consider a DS/CDMA system with three active users, each of which is assigned a length 127 Gold code as its signature code. The channel is normalized such that $E[|c_{kl}|^2] = 1$, for $1 \le k \le K$ and $1 \le l \le L$. In the following simulations, the multipath number is assumed to be two, i.e., $L = 2$, and three classes of channel estimation mismatch, i.e., the mismatch of timing, phase estimation and amplitude estimation, are examined separately. The SNR in the figures is the received signal to noise ratio of user 1.

In the first example, the performances of the RDD and the MDD in the presence of timing mismatch are examined. For simplicity, it is assumed that $\tau_{11} = \tau_{21} = \tau_{31}$, $\tau_{12} = \tau_{22} = \tau_{32}$ and that all the timing is mismatched equally, i.e., $\hat{\tau}_{11} = \hat{\tau}_{21} = \hat{\tau}_{31}$, $\hat{\tau}_{12} = \hat{\tau}_{22} = \hat{\tau}_{32}$ and $\hat{\tau}_{k1} - \tau_{k1} = \hat{\tau}_{k2} - \tau_{k2}$ $(1 \le k \le 3)$. The BER of user 1 of the RDD and the MDD with interferers received with equal and 10 dB more power are given in Figs. 1 and 2, respectively. The results show that, although the RDD has a lower error probability than the MDD when the channel estimation is perfect, it is more sensitive to timing mismatch than the MDD. As the amount of timing mismatch increases, the error probability and the near-far resistance of the RDD deteriorate more rapidly than those of the MDD.

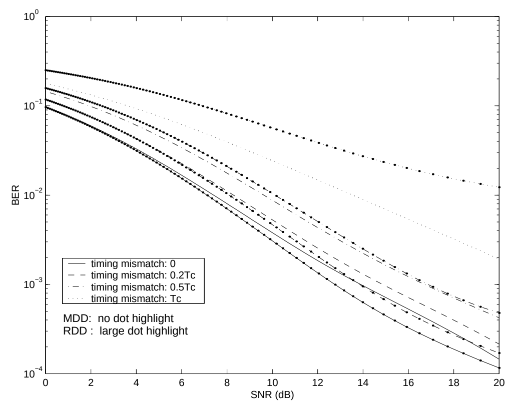

Fig. 1. BER with mismatched timing and interferers received with equal power (L=2)

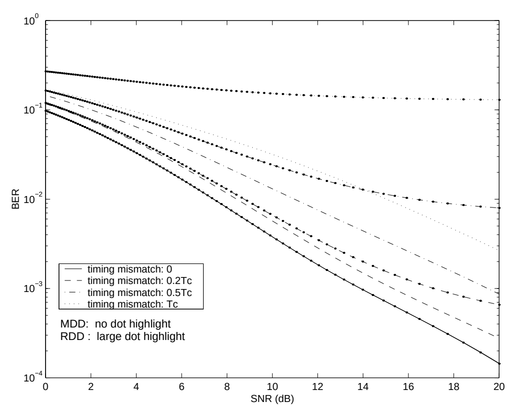

Fig. 2. BER with mismatched timing and interferers received with 10 dB more power (L=2)

In the second example, the performances of both detectors in the presence of phase estimation mismatch are considered. It is assumed that $\alpha_{11} = \alpha_{21} = \alpha_{31}$, $\alpha_{12} = \alpha_{22} = \alpha_{32}$ and that all the phase is mismatched equally, i.e., $\hat{\alpha}_{11} = \hat{\alpha}_{21} = \hat{\alpha}_{31}$, $\hat{\alpha}_{12} = \hat{\alpha}_{22} = \hat{\alpha}_{32}$ and $\hat{\alpha}_{k1} - \alpha_{k1} = \hat{\alpha}_{k2} - \alpha_{k2}$ $(1 \le k \le 3)$. The BER of the RDD and the MDD with interferers received with equal

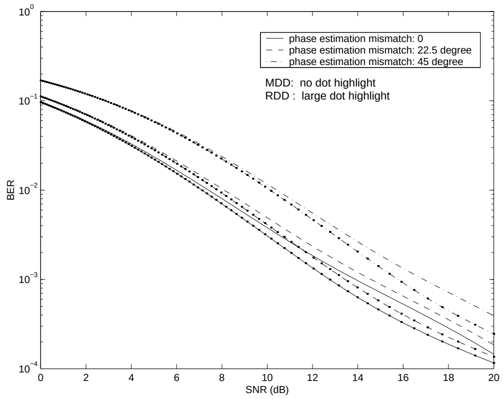

Fig. 3. BER with mismatched phase estimation and interferers received with equal power (L=2)

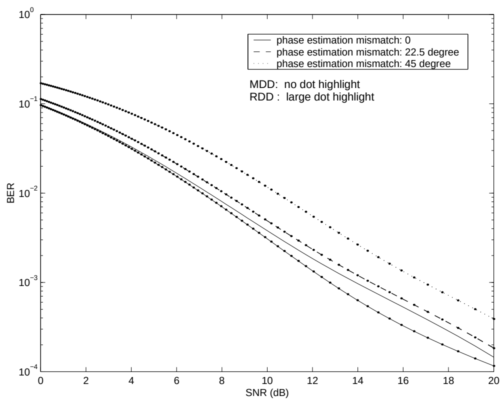

Fig. 4. BER with mismatched phase estimation and interferers received with 10 dB more power (L=2)

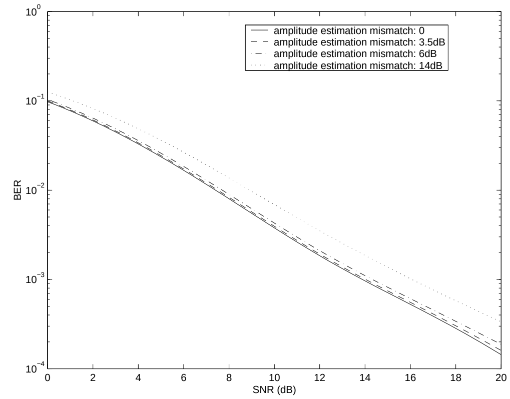

Fig. 5. BER of RDD with mismatched amplitude estimation and interferers received with equal power (L=2)

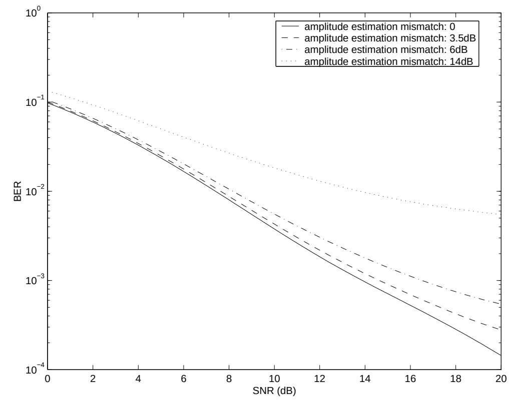

Fig. 6. BER of RDD with mismatched amplitude estimation and interferers received with 10 dB more power (L=2)

and 10 dB more power are depicted in Figs. 3 and 4, respectively. It is shown that the near-far resistance of the RDD degrades faster than that of the MDD as the amount of phase mismatch increases. However, both of them can still maintain a good performance even when the phase estimation mismatch is quite large.

It is easily seen from (21) that the MDD does not need to estimate amplitude in the process of demodulation and therefore has no amplitude estimation mismatch problem. As to the RDD, Figs. 5 and 6 give its performance in the presence of amplitude estimation mismatch. Here it is assumed that the received signal amplitude of user 1 is estimated without error and that all the interfering signals' amplitudes are mismatched to the same extent in the sense of dB. Similarly, the performance of the RDD becomes worse as the amplitude estimation mismatch becomes larger.

IV. Conclusion

In this paper, we studied the probability of error and the near-far resistance of the RDD and the MDD in mismatched frequency-selective multipath Rayleigh fading channel. Numerical results show that, although the RDD has a better performance than the MDD based on perfect channel estimation, it is more sensitive to channel estimation error. As the amount of the channel estimation mismatch increases, the performance of the RDD degrades more rapidly than that of the MDD.

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