Joint iterative Turbo decoding and estimation of correlated Rayleigh fading channel

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Abstract

Turbo codes, also known as parallel concatenated recursive systematic convolutional codes, have been shown to perform near the Shannon capacity limit in AWGN channels. In mobile communication cases, the channel has a significant correlation. In this paper, we propose an iterative joint correlated Rayleigh fading channel estimation and turbo decoding procedure. The detector (estimator/decoder) is a soft-in soft-out process. The decoder consists of the BJCR-MAP algorithm and the channel estimator is based on the MMSE criterion. The channel estimation process does not require pilot symbols and only the channel statistics are supposed known. The performance of the proposed method is evaluated by Monte-Carlo simulation for rate 1/2 and 1/3 turbo code and compared for three normalized fading rates.

I. Introduction

Turbo codes introduced in [1] can achieve performance close to the Shannon capacity limit in both the AWGN channel and the Rayleigh flat-fading channel with perfect knowledge of Rayleigh process. However, in practice, the channel side information (CSI) is not available. So, a natural approach is to estimate the channel impulse response and to use the estimated values to compute the channel reliability factor required by the MAP algorithm.

Recently, turbo codes performance over Rayleigh fading channels has been studied in several publications [2–6]. Assuming perfect knowledge of the fading process parameters, Barbulescu [7] has considered the Rayleigh channel as an AWGN one conditioned on the known fading amplitude. Hall and Wilson [2] proposed a turbo decoding scheme when the phase is known, but not the amplitude. This scheme calculates the *pdf* of the received signal by averaging the fading amplitude process over all possible values. With a high Doppler rate and low SNR it is not reasonable to substitute the real fading coefficient by the average value. The papers [8,9] have investigated the effects of the mismatch of the channel reliability on the turbo decoding performance. In [4], Frenger assumes that the channel estimate is available and derives the channel reliability factor but he does not discuss the problem of channel estimation. Some studies used pilot symbols to estimate the fading process parameters [3,6]. In [5], the authors consider PSK modulation and propose a non-pilot-aided joint iterative phase estimation and data decoding.

In this paper, we propose an iterative joint correlated Rayleigh fading channel estimation and BJCR-MAP decoding. The proposed detector scheme called "turbo-detector" is composed of the concatenation of a channel estimation block and a MAP decoder. The turbo-detector is a soft-in/soft-out (SISO) algorithm and the information passed between channel estimator and turbo decoder takes soft values. Channel estimation is based on the minimum mean squared error (MMSE) criterion and does not require a training sequence.

The outline of this paper is as follows. Section II includes a brief presentation of the system and channel models and the turbo decoding algorithm. We then propose, in section III, the iterative estimation/decoding technique. Simulation results are shown in section IV and conclusions are made in section V.

II. System Model

Let $\{d_k\}$ an information data stream of length L_d encoded using $R = 1/r$ rate turbo code and BPSK modulated. The encoded and modulated codeword sequences $\mathbf{c_k} = [\mathbf{c_k^0}, \mathbf{c_k^1}, \ldots, \mathbf{c_k^{r-1}}]$ are then interleaved and transmitted over a correlated Rayleigh flat-fading channel with AWGN as shown in Figure 1. The interleaver $\Pi(.)$ is used to randomize the transmitted symbols in order to combat the effects of the channel correlation.

Let $\mathbf{y}$ be the received sequence of length $N = L_d/R + L_0$ (L_0 : tail) whose elements y_k^l are

$$y_k^l = g_k x_k^l + n_k^l = g_k(2c_k^l - 1) + n_k^l \qquad (1)$$

where n_k^l is an AWGN with variance $\sigma^2 = N_0/2$ and g_k is the multiplicative distortion of the flat-fading channel.

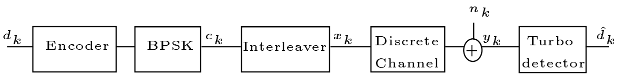

Fig. 1. Transmission scheme.

The channel gain is supposed to be a correlated complex gaussian process with zero mean, variance σ_g^2 and Doppler spread f_d. Its auto-correlation $R_g(l) = \rho_l$ function can be written as [10]

$$\rho_l = E\{g_k g_{k-l}^*\} = \sigma_g^2 J_0(2\pi f_d l T_s) e^{j2\pi f_d l} \qquad (2)$$

At the receiver, the well-known BJCR-MAP decoding algorithm computes the a-posteriori probabilities $\Pr\{d_k = 1|\mathbf{y}\}$ and $\Pr\{d_k = 0|\mathbf{y}\}$ and its output is the log likelihood ration (LLR) given by

$$\begin{aligned}\Lambda(d_n) &= \ln \frac{\Pr\{d_n = 1|\mathbf{y}\}}{\Pr\{\mathbf{d_n} = \mathbf{0}|\mathbf{y}\}} \\ &= \ln \frac{\Pr\{\mathbf{y}|\mathbf{d_n} = \mathbf{1}\}}{\Pr\{\mathbf{y}|\mathbf{d_n} = \mathbf{0}\}} + \ln \frac{\Pr\{d_n = 1\}}{\Pr\{d_n = 0\}} \\ &= \Lambda_e(d_n) + \Lambda_a^E(d_n) \qquad (3)\end{aligned}$$

which can be written as the sum of the a priori information and the extrinsic information. For a known channel state cases, the LLR is given by [1]

$$\begin{aligned}\Lambda(d_n) &= 4.g_n.\frac{E_s}{N_0}.y_n + \Lambda_a^E(d_n) \\ &= L_n.y_n + \Lambda_a^E(d_n) \qquad (4)\end{aligned}$$

where E_s is the energy per symbol and L_n is called the channel reliability which depends on the SNR and channel gain. So, for system without CSI, we need to estimate channel parameters.

III. Iterative joint Turbo decoding and channel estimation

In this section, we present a joint turbo decoding and channel estimation method. For the algorithm that we propose, the channel estimation and the decoding are performed through an SISO iterative process as is shown in Figure 2. The decoding is carried out by the BJCR-MAP algorithm as in [1]. In the first iteration, we do not estimate the channel, and the detector uses only the channel output to estimate the transmitted symbols. So, the channel reliability is calculated by averaging the fading process over all possible values such as in [2].

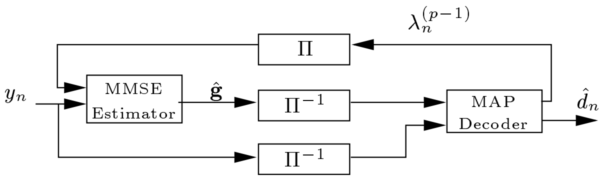

Fig. 2. Turbo detector structure.

For an iteration ($p > 1$), one approach has been to use the estimation of the sequence $\{x_k\}$, given by the last iteration and denoted by $\{\hat{x}_k^{(p-1)}\}$, to estimate the channel vector $\mathbf{g} = [\mathbf{g_1}, \ldots, \mathbf{g_N}]$. Instead of using $\{\hat{\mathbf{x}}_k^{(p-1)}\}$, our estimation procedure uses a soft decision $\lambda_k^{(p-1)} = \mathrm{E}\{x_k^{(p-1)}\}$ which is the mean value of x_k

$$\begin{aligned}\lambda_k^{(p-1)} &= \Pr\{x_k = +1\}.1 + \Pr\{x_k = -1\}.(-1) \\ &= \tanh\left(\frac{\Lambda(x_n)}{2}\right) \qquad (5)\end{aligned}$$

Knowing that the sequence $\{\mathbf{x_k}\}$ is an interleaved version of $\{\mathbf{c_k}\}$, we have $\Lambda(x_n) = \Pi(\Lambda(c_n))$. Hence, the input of the channel estimator module is an interleaved version of the decoder output and its output is the vector $\hat{\mathbf{g}}^{(p)}$ obtained by the minimization of the mean squared error

$$\mathrm{E}\{|\hat{\mathbf{g}}^{(p)} - \mathbf{g}|^{\mathbf{2}}\} \qquad (6)$$

Let us denote $\mathbf{R_g}$ the Toeplitz correlation matrix of the fading process, $\mathbf{R_n} = \sigma^{\mathbf{2}}\mathbf{I^N}$ the noise correlation matrix and $\hat{\Lambda}_{\mathbf{p-1}} = \mathbf{Diag}(\lambda_{\mathbf{k}}^{(\mathbf{p-1})})$. Given the received sequence $\mathbf{y}$, the fading process has a gaussian distribution $f_{\mathbf{G}|\mathbf{Y}}(\mathbf{g}|\mathbf{y})$ with the following parameters

$$\mathrm{E}[\mathbf{g}|\mathbf{y}] = \mathbf{R_g}\hat{\Lambda}_{\mathbf{p-1}}^{\mathbf{t}}\left(\hat{\Lambda}_{\mathbf{p-1}}\mathbf{R_g}\hat{\Lambda}_{\mathbf{p-1}}^{\mathbf{t}} + \mathbf{R_n}\right)^{-1}\mathbf{y}$$

$$R_{\mathbf{g}|\mathbf{y}} = \mathbf{R_g} - \mathbf{R_g}\hat{\Lambda}_{\mathbf{p-1}}^{\mathbf{t}}\left(\hat{\Lambda}_{\mathbf{p-1}}\mathbf{R_g}\hat{\Lambda}_{\mathbf{p-1}}^{\mathbf{t}} + \mathbf{R_n}\right)^{-1}\hat{\Lambda}_{\mathbf{p-1}}\mathbf{R_g}$$

Therefore, the MMSE estimator $\hat{\mathbf{g}}^{(p)}$ is the conditional mean given by

$$\hat{\mathbf{g}}^{(P)} = \mathbf{R_g}\hat{\Lambda}_{\mathbf{p-1}}^{\mathbf{t}}\left(\hat{\Lambda}_{\mathbf{p-1}}\mathbf{R_g}\hat{\Lambda}_{\mathbf{p-1}}^{\mathbf{t}} + \mathbf{R_n}\right)^{\mathbf{-1}}\mathbf{y} \qquad (7)$$

We note that the matrix $\mathbf{R_g}\hat{\Lambda}_{\mathbf{p-1}}^{\mathbf{t}}\left(\hat{\Lambda}_{\mathbf{p-1}}\mathbf{R_g}\hat{\Lambda}_{\mathbf{p-1}}^{\mathbf{t}} + \mathbf{R_n}\right)^{\mathbf{-1}}$ is the Wiener filter.

The estimated vector $\hat{\mathbf{g}}^{(p)}$ is then interleaved and passed to the decoder. Having the channel estimation, the

BCJR-MAP decoding algorithm processes as in the case of CSI and so, the channel reliability is given by

$$L_n = 4|\hat{g}_n^{(p)}|\frac{E_s}{N_0}y_n \tag{8}$$

After P iterations of turbo-decoding and channel estimation, the soft output of the turbo decoder is thresholded to yield an estimate of the information sequence $\hat{d}_k$. The advantage of this procedure is that the channel estimator uses the redundancy information introduced by the channel encoder. In fact, for classical receiver structure, estimation and decoding process are disjoint and each one is terminated by a hard decision. In this situation the global system performance is suboptimal because we do not use the totality of the available information.

IV. Simulation Results

To analyze the performance of the proposed receiver structure, computer simulations have been carried out using both a rate 1/3 turbo code and a punctured code of rate 1/2. We have used a turbo code of constraint length $K = 3$ with a generator matrix $(1, 5/7)$ in octal form and a random encoder interleaver. The data frame size is $N = 1024$ and the decoding algorithm uses 5 iterations. The transmission channel was assumed Rayleigh flat-fading and three normalized fade rates were considered, $f_D T_s \in \{.002, .02, .2\}$. As in [6], the punctured code (rate 1/2) is obtained by deleting the even indexed parity bits from the first RSC encoder and the odd indexed from the second RSC encoder.

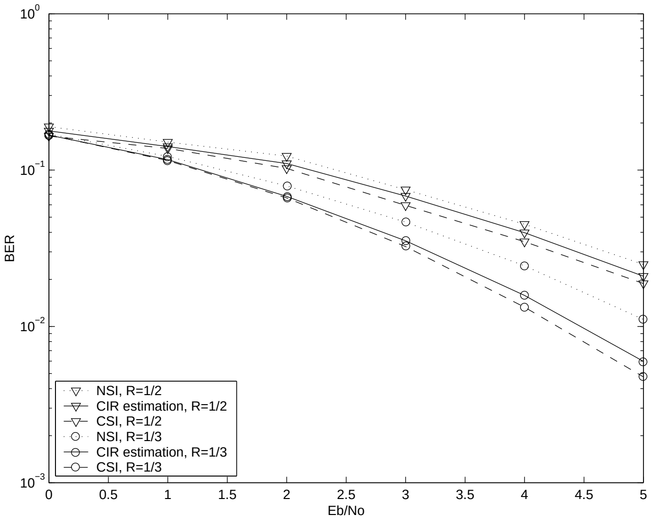

Fig. 3. BER performance over Rayleigh fading channel with $f_D T_s = 0.002$.

Simulation results for normalized fade rates $f_D T_s = .002, .02$ and $.2$ are respectively shown in Figure 3, 4 and 5. Each figure gives the curves of BER vs. E_b/N_0 for two coding rates $R = 1/2$ and $R = 1/3$ and three detection cases. In fact, we have simulated 3 detection scenarios: system with CSI, system without CSI (NSI: no side information) and the case where the channel is estimated.

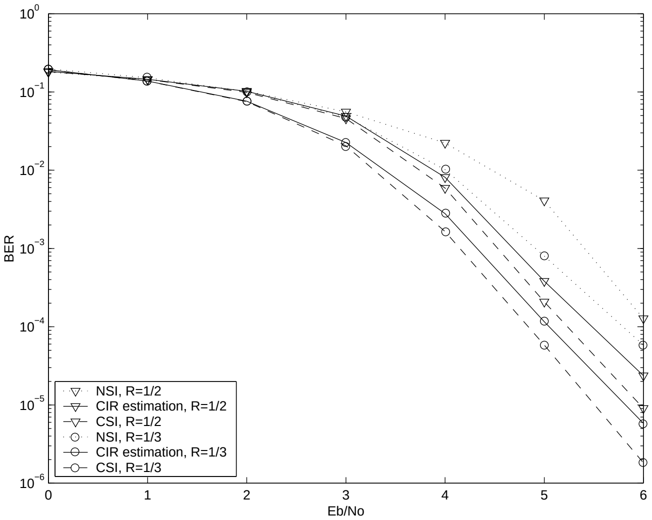

Fig. 4. BER performance over Rayleigh fading channel with $f_D T_s = 0.02$.

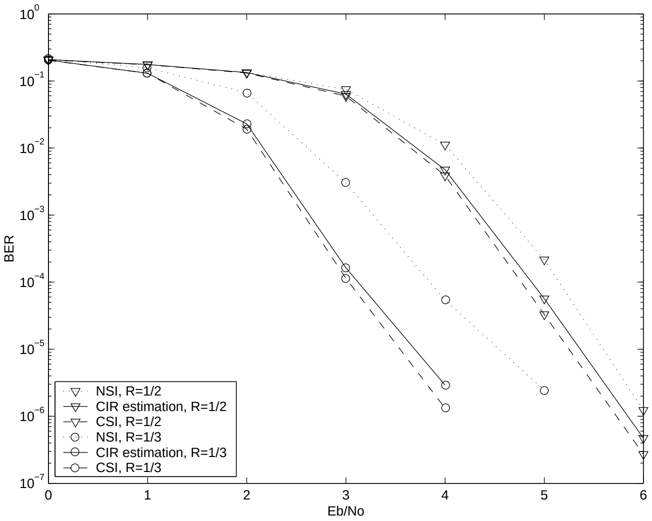

Fig. 5. BER performance over Rayleigh fading channel with$f_D T_s = 0.2$.

Naturally, the rate 1/3 turbo code is offers a better performance than the rate 1/2 and the performances of

CSI cases (dashed line) are better than those of the other cases. Nevertheless, we can see that the difference between the performances of the two codes is less important if $f_D T_s$ increases. Indeed, for a BER of 10^{-3}, the discrepancy is about 0.5 dB when $f_D T_s = 0.02$ and 1.5 dB when $f_D T_s = 0.2$.

The figures show that the joint turbo decoding and channel estimation procedure improves the system performance. The results obtained when channel estimation is performed (dotted line) approach that of the CSI system. It can be seen in Figure 3 that the discrepancy between the CSI and NSI (solid line) system performance for a BER of 10^{-4} is about 1 dB where $R = 1/2$ and $f_D T_s = 0.02$.

By looking at the figures, we notice that the performances are better when the normalized fade rates $f_D T_s$ increase. In fact, the Doppler rate decreases as the correlation between the transmitted signals over the Rayleigh channel increases and then the probability of burst errors increases.

V. Conclusion

We have successfully developed a turbo detector which joint data decoding and channel estimation. The proposed method does not use a training sequence and requires only knowledge of the fading channel's correlation function. An MMSE estimator was proposed to estimate the channel coefficients and the the BCJR-MAP algorithm has been used for turbo decoding. The decoder uses the channel and estimator outputs. The soft-decision of the decoder is then fed back to the channel estimator. After P iterations of turbo-decoding and channel estimation, the soft output of the turbo decoder is thresholded to yield an estimate of the information sequence. It has been shown that the information exchanging between the decoder and the estimator improves the system performance and the channel estimation plays an important role in the BCJR-MAP decoding. Simulations for two turbo code rates and three fading rates indicate that the detection procedure leads to a substantial improvement in the system performance over a correlated Rayleigh fading channel.

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