Iterative Multiuser Decoding of Narrowband Signals in Fading Channels

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Abstract—
In this paper we investigate multiuser detection techniques for coded narrowband signals in fading channels. In [1], Moher proposed a suboptimal multiuser detector and its performance is demonstrated for the AWGN channel. In this paper, this multiuser detector is modified for fading channels, such as the Rayleigh fading channel model. The fading channels described and simulated are flat narrowband channels. The receiver is assumed to be synchronous.

I. Multiuser Channel Model

A. Discrete Time Synchronous Model

Multiuser transmission means that K users share one resource for transmission of data. The shared resource can be the spectrum, time, or power allocations. The common code multiplex technique CDMA is used where it makes sense to have a concrete example, for motivating the model equation and for giving an example how to calculate the crosscorrelations. The model itself can be used with all types of multiuser transmission and is not specific for CDMA.

In a CDMA system, every user k is assigned a unique signature waveform $s_k(t)$ known by the receiver. If the length of the waveforms is not larger than the symbol duration T, there is no ISI and a synchronous model can be used. That is, the received signal $y(t)$ depends only on the mapped bits $b_i, 1 \leq i \leq K$ of the actual point of time. If the used channel is an AWGN channel the difference between the both signal after the channel $y(t)$ and before the channel $s(t)$ is a white Gaussian distributed noise process:

$$y(t) = s(t) + n(t) \tag{1}$$

where $n(t)$ represents white Gaussian noise with variance σ^2. The signal at the input of the receiver can be described for a symbol duration T as follows:

$$y(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + n(t), \quad t \in [0, T] \tag{2}$$

A_k is the received amplitude of the k^{th} user's signal.

If the signature waveforms were orthogonal, we could achieve the same performance as in the single user case just by using a bank of matched filters on $y(t)$ and handling the received samples like in the single user scenario. If the signature waveforms are not exactly orthogonal,

$$\rho_{ij} = \langle s_i, s_j \rangle = \int_0^T s_i(t)\, s_j(t)\, dt \tag{3}$$

is not zero. ρ_{ij} is the crosscorrelation for the users i and j. For every single user k, a specific matched filter with output y_k is used, matched to the signature waveform of this specific user:

$$y_k = \int_0^T y(t) s_k(t)\, dt = A_k b_k + \sum_{j \neq k} A_j b_j \rho_{jk} + n_k, \tag{4}$$

with $n_k = \int_0^T n(t)\, s_k(t)\, dt$. Equation (4) can be described in vector form:

$$\mathbf{y} = \mathbf{RAb} + \mathbf{n} \tag{5}$$

The vector $\mathbf{y}$ of length K has all $y_k, 1 \leq k \leq K$ as entries. The $K \times K$ matrix $\mathbf{R}$ has the cross correlations ρ_{ij} as entries. The received amplitudes are represented by $\mathbf{A}$. It's a $K \times K$ matrix only having non zero entries A_k in the diagonal. The vector $\mathbf{b}$ of length K has all $b_k, 1 \leq k \leq K$ as entries. The vector $\mathbf{n}$ of length K has all $n_k, 1 \leq k \leq K$ as entries. The noise vector has the properties $E\left\{\mathbf{n} \cdot \mathbf{n}^H\right\} = \mathbf{R}$ and $\mathbf{n} = \sqrt{\mathbf{R}} \cdot \mathbf{n}_0$ with $\mathbf{n}_0$ is the vector of the complex Gaussian white noise values before matched filtering as introduced in [6]. A simple example for the crosscorrelation matrix $\mathbf{R}$ is the K-symmetric channel. In this case, all couples of signature waveforms have the same crosscorrelation ρ.

B. Rayleigh Fading Channel

To model users in a mobile multiuser environment, we use flat correlated Rayleigh channels with a normalized maximum Doppler frequency $f_{D,max}T_S$ of 0.01. The received amplitudes are fading due to the time variant interference of the echos arriving at the receiver:

$$\mathbf{A} = \mathbf{A}[i], 1 \leq i \leq N \tag{6}$$

with N being the number of transmitted symbols. We consider narrowband flat fading. The difference to the single user case is that we have to handle with K channel coefficients for every symbol duration instead of only 1 because there are K users. Assuming that we use a BPSK transmission with bit energy 1 the entries of $\mathbf{A}[i]$ are exactly the coefficients describing the Rayleigh fading. The phase

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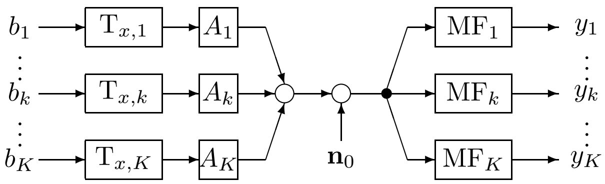

Fig. 1. Synchronous time discrete model for multiuser transmission over Rayleigh channel.

of the fading can be neglected as we assume a coherent receiver. The illustration of the synchronous discrete time model for a coherent transmission over one common channel can be seen in figure 1.

II. Multiuser Decoding

A. Decorrelating Detector

The idea of using single user matched filter and making hard decisions on the Matched Filter (MF) outputs works fine if the correlation between the signals from the different users is low. But for higher crosscorrelations values, e.g. $\rho \geq 0.5$, the interfering noise is stronger and leads to a higher BER. One well-known method to give the receiver information about the crosscorrelations is the decorrelating detector. The principle of the decorrelating detector is to use this knowledge to decorrelate the signals of the users. Equation (5) describes the MF output samples. If we assume no noise, then (5) simplifies to $\mathbf{y} = \mathbf{RAb}$. If we left-multiply the right side with the inversion of the cross correlation matrix $\mathbf{R}^{-1}$, we get exactly the sent information vector $\mathbf{b}$ but with different gains. In this noiseless case a hard decision on these values would mean an error-free detection. If we admit a noise vector $\mathbf{n}$, the left-multiplication leads to coloured noise:

$$\mathbf{y}' \quad = \quad \mathbf{R}^{-1} \cdot (\mathbf{RAb} + \mathbf{n}) = \mathbf{Ab} + \mathbf{R}^{-1}\mathbf{n} \qquad (7)$$

with the received vector $\mathbf{y}'$ being the same as the sent $\mathbf{b}$ but with different amplitudes plus additive coloured Gaussian noise.

B. Moher's Iterative Multiuser Detection Algorithm

In the multiuser case, the distribution of the received sample with a given sent symbol transmitted over an AWGN channel can be described using a multivariate Gaussian distribution:

$$\Theta[\mathbf{y}|\mathbf{b}] = \frac{|\mathbf{R}|}{(2\pi)^{\frac{K}{2}}\,\sigma^K} e^{-\frac{(\mathbf{y}-\mathbf{Rb})^H \mathbf{R}^{-1}(\mathbf{y}-\mathbf{Rb})}{2\sigma^2}} \qquad (8)$$

Implementing Moher's algorithm [1] means giving the receiver information about the crosscorrelation values and also about the channel conditions. Assuming a multivariate AWGN channel, we know the distribution of the received samples for a given vector of bits $\mathbf{b}$ sent by the K users at time i as given by (8). The evaluation of this distribution requires the knowledge of the crosscorrelation matrix $\mathbf{R}$ and the variance of the noise, that is we must know how the received samples of the users are correlated and the statistics of the AWGN noise. The first step of Moher's algorithm is to calculate the initial a priori distribution assuming uniformly distributed channel bits:

$$q_0[\mathbf{b}] = \Theta[\mathbf{b}|\mathbf{y}] = \frac{\Theta[\mathbf{y}|\mathbf{b}]\,\Theta[\mathbf{b}]}{\Theta[\mathbf{y}]}. \qquad (9)$$

As the receiver does not know $\mathbf{b}$, equation (9) is calculated for every hypotheses $\tilde{\mathbf{b}}$.

$$q_0\left[\tilde{\mathbf{b}}\right] = \Theta\left[\mathbf{y}|\tilde{\mathbf{b}}\right]\Big|_{\mathrm{norm}} \qquad (10)$$

The second step consists in computing the marginal distributions $q_0[b_k]$, $1 \leq k \leq K$:

$$q_0\left[\tilde{b}_k = 0\right] = \sum_{\tilde{\mathbf{b}}:\tilde{b}_k=0} q_0\left[\tilde{\mathbf{b}}\right] \qquad (11)$$

With these distributions we have the probability of a sent symbol 0 (or 1 respectively) for every user k at time i given the received signals of all users. The sum is calculated over all hypotheses with a 0 as the hypothesis for the sent symbol of user k. Then the sum consists of 2^{K-1} addends. In our simulations, we calculate the marginal distributions for a sent symbol 0: when a marginal distribution is used, it means the marginal distribution for a sent 0 symbol.

These probabilities are used as input for the BCJR algorithm [2] instead of the received samples (actually we can use exactly the same algorithm with the corresponding Log Likelihood Ratios (LLR)). The third step is a decoding step which gives the output distribution:

$$p_j\left[\tilde{b}_k\right] = q_{j-1}\left[\tilde{b}_k\right] p_{e,j}\left(\tilde{b}_k\right)\Big|_{\mathrm{norm}} \qquad (12)$$

With equation (12) the decoding is interpreted as a function that adds information. We can split the result into an intrinsic part already obtained before the decoding $q_{j-1}\left[\tilde{b}_k\right]$ and an extrinsic part the decoding process has created $p_{e,j}\left(\tilde{b}_k\right)$. The output of the decoder again is a normalized distribution.

The last step of Moher's algorithm consists in closing the loop for iterative decoding. We assumed above a uniform a priori distribution because we had no information about the distribution of the channel bits. In the iterative mode, we calculate the probability of every $\tilde{\mathbf{b}}$ at every instant i after the decoding with the BCJR algorithm and return to the first step of the algorithm using the new distribution $p_j\left(\tilde{\mathbf{b}}\right)$ instead of the uniform distribution $\Theta\left(\tilde{\mathbf{b}}\right)$. For the following iteration steps, the latter distribution $p_j\left(\tilde{\mathbf{b}}\right)$ is replaced by the new one $p_{j+1}\left(\tilde{\mathbf{b}}\right)$, and so on. The calculation of $p_j\left(\tilde{\mathbf{b}}\right)$ is made with the assumption that the hypotheses of the single users b_k are statistically independent:

$$p_j\left(\tilde{\mathbf{b}}\right) = \prod_{k=1}^{K} p_j\left(\tilde{b}_k\right) \qquad (13)$$

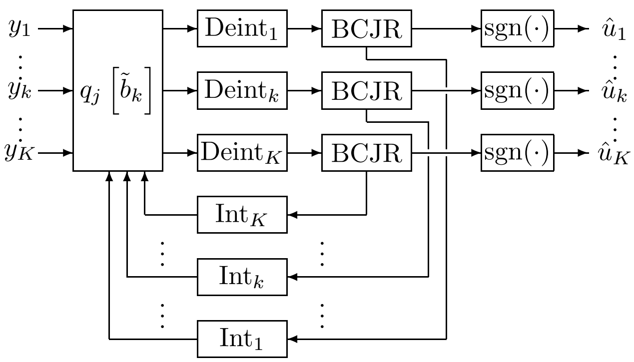

Fig. 2. Moher's multiuser detection algorithm in closed loop (iterative) mode.

This can be achieved using an interleaver with a sufficiently large interleaving depth after the encoder at the transmitter and a deinterleaver before the BCJR algorithm decoder at the receiver. Every user has to use a different interleaver, as shown in figure 2. For the simulations done here, only the extrinsic part of the distribution $p\left(\tilde{b}_k\right)$ is fed back:

$$p_{e,j}\left(\tilde{\mathbf{b}}\right)=\prod_{k=1}^{K} p_{e,j}\left(\tilde{b}_k\right) \tag{14}$$

This leads to an improvement of the BER performance as the number of iterations increases. Feeding back both the extrinsic $p_{e,j}\left(\tilde{b}_k\right)$ and the intrinsic $q_{j-1}\left(\tilde{b}_k\right)$ information, which is equivalent to feeding back $p_j\left(\tilde{b}_k\right)$ itself, leads to an improvement in comparison to the open loop case but is not as good as feeding back only the extrinsic information. As the used MAP decoder is optimal for a single user transmission, the distribution for user k can not be improved after the first decoding without using distributions of the other users. Therefore, the extrinsic information $p_{e,j}\left(\tilde{b}_k\right)$ can be omitted in the calculation of $q_j\left(\tilde{b}_k\right)$ in the next interation:

$$\tilde{q}_j\left(\tilde{\mathbf{b}}\right)=\Theta\left(\mathbf{y}|\tilde{\mathbf{b}}\right)\cdot p_{e,j}\left(\tilde{\mathbf{b}}\right)\Big|_{\text{norm}} \tag{15}$$

The algorithm runs for a number of iterations. After the last decoding step the output of the decoder is not interleaved. A hard decision on the symbols is then made from the marginal distribution at the output of the decoder.

C. Comparison of Decorrelating Detector and Moher's Algorithm

Both the decorrelating detector and Moher's algorithm require information about the crosscorrelation of the users. But Moher's algorithm also uses information about the channel. This leads to the conclusion that the latter receiver should be more power efficient. Figure 3 shows the result of a 2 user transmission over an AWGN channel using a rate $\frac{1}{2}$ constraint length 5 convolutional code in closed loop configuration. The K-symmetric channel model is used. The curve of the BER goes to the single user curve

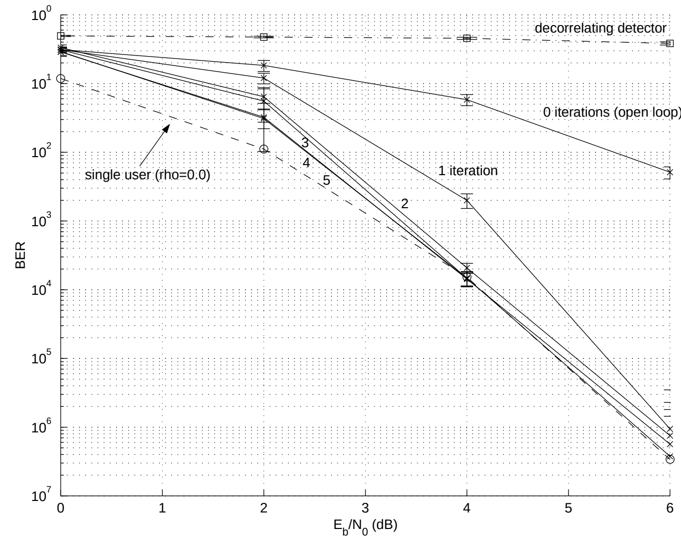

Fig. 3. Coded transmission over AWGN channel using $r=\frac{1}{2}$ convolutional code, decorrelating detector and Moher's algorithm, $\rho=0.9$, and $K=2$ users.

as the numbers of iterations increase. Not more than two or three iterations are necessary to achieve good results. For SNR values of $4dB$ and higher there is no significant differences between the single user case and the multiuser case using Moher's algorithm with three iterations. Performing more iterations does not lead to noticeably better results.

III. Decoding Performance in Rayleigh Fading

The multiuser detection algorithm uses information about the channel the data is transmitted over. This means that we have to change the distribution $\Theta\left[\mathbf{y}|\mathbf{b}\right]$ so that the channel is described correctly.

The AWGN channel using multiuser transmission can be described with (8):

$$\Theta\left[\mathbf{y}|\mathbf{b}\right]=\frac{|\mathbf{R}|}{(2\pi)^{\frac{K}{2}}\,\sigma^K}e^{-\frac{(\mathbf{y}-\mathbf{R}\mathbf{b})^H\mathbf{R}^{-1}(\mathbf{y}-\mathbf{R}\mathbf{b})}{2\sigma^2}} \tag{16}$$

The discrete time synchronous model as matrix equation is described with (5):

$$\mathbf{y}=\mathbf{R}\mathbf{A}\mathbf{b}+\mathbf{n} \tag{17}$$

We already introduced the matrix $\mathbf{A}$ describing the different power levels of the single users. As we assume a coherent receiver, the phase of the channel coefficients of the Rayleigh fading channel can be omitted and the fading can be modeled by multiplication of the signals of the users with real values. The channel coefficients represent only the amplitudes since the phases are omitted due to the coherent reception. The matrix $\mathbf{A}$ can be used to describe both the different amplitudes of the users and the fading due to the channel. If we assume equal power users and coherent receivers, the matrix $\mathbf{A}$ holds the amplitudes of the fading channel coefficients. This means that (5) can be used to describe the multiuser transmission over a Rayleigh

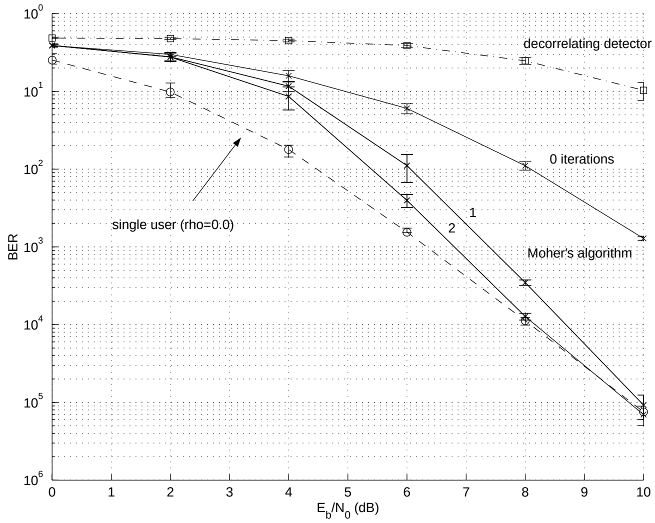

Fig. 4. Coded transmission over Rayleigh channel using $r = \frac{1}{2}$ convolutional code, decorrelating detector and Moher's algorithm, $K = 2$ users, and a crosscorrelation of $\rho = 0.9$.

channel. The only change that has to be made in (8) to describe the distribution of the Rayleigh channel instead of the AWGN channel is to replace the crosscorrelation matrix $\mathbf{R}$ with the matrix product $\mathbf{RA}$ so that we get the distribution:

$$\Theta\left[\mathbf{y}|\mathbf{b},\mathbf{A}\right] = \frac{|\mathbf{R}|}{(2\pi)^{\frac{K}{2}}\ \sigma^K} e^{-\frac{(\mathbf{y}-\mathbf{RAb})^H \mathbf{R}^{-1}(\mathbf{y}-\mathbf{RAb})}{2\sigma^2}} \tag{18}$$

The simulation result for two users and a crosscorrelation of $\rho = 0.9$ is shown in figure 4. All users transmit over the same Rayleigh channel, that is, the time variant channel coefficient is the same for all users. So A is a time dependent matrix which can be described by the product $f_i\mathbf{I}$ for one point of time, where f_i is the channel coefficient for sample i and $\mathbf{I}$ is an identity matrix with the same size as $\mathbf{A}$. In comparison to the decorrelating detector Moher's algorithm leads to significantly better results and with only a few iterations nearly the performance of the single user case can be achieved for an SNR of $4dB$ or higher. The result for five users is shown in figure 5.

IV. Conclusion

In this paper we investigated the bit error rate performance of multiuser detection techniques with and without channel coding on AWGN and Rayleigh channels. Moher's suboptimal iterative multiuser detection algorithm was introduced and the performance in open loop mode as well as in closed loop mode (iterative mode) on the AWGN channel was compared to the performance of the suboptimal decorrelating detector. Moher's algorithm was then modified to adapt it for Rayleigh narrowband fading channels. The performance of the modified algorithm was presented and it is shown that even with the common Rayleigh channel model, single user performance could be reached after a few iterations of Moher's algorithm provided that the signal to noise ratios are sufficiently large.

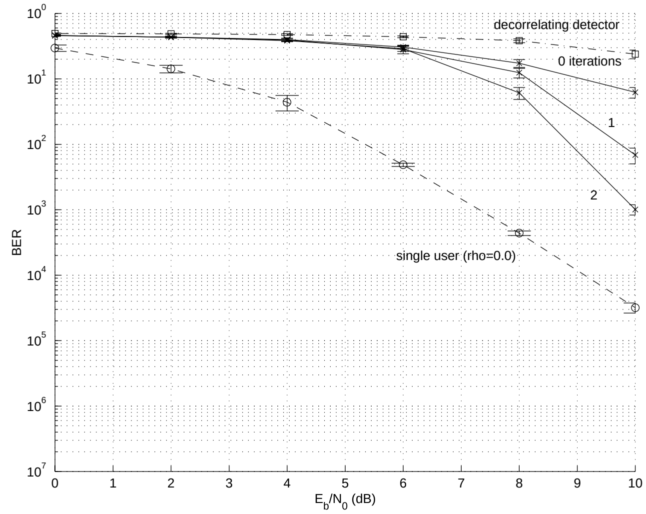

Fig. 5. Coded transmission over Rayleigh channel using $r = \frac{1}{2}$ convolutional code, decorrelating detector and Moher's algorithm, $K = 5$ users, and a crosscorrelation of $\rho = 0.9$.

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