# Performance of an Improved Modified MMSE Receiver for CDMA Systems in Rayleigh Fading†

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***Abstract* — In this paper, the bit error rate of a recently proposed modified minimum mean squared error (MMSE) detector in Rayleigh flat fading channels is improved. It is also shown that the new detector is less sensitive to changes in the number of users. This improved detector is then adapted for use in a Rayleigh faded frequency-selective channel. An asynchronous binary phase shift keying (BPSK) direct sequence code division multiple access (DS/CDMA) system is assumed, with pilot symbols inserted into the data stream, for channel estimation.**

***Index Terms* — adaptive MMSE receiver, interference suppression, fading, CDMA, adaptive algorithms**

## I. Introduction

Interference suppression is a useful technique for improving the signal quality of a desired user in a DS-CDMA system, by suppressing the interference due to other users. The linear minimum mean squared error (MMSE) detector uses a linear transformation to remove as much of the interference due to other users as possible. However, it requires extensive information about the interfering users, as well as very frequent updates. This is not suitable for the mobile user, where information about other users is not known, and signal processing capabilities are limited. The MMSE detector easily leads to an adaptive version, where only information about the desired user is known [1], or the desired user plus a training or pilot sequence [2].

The MMSE criterion has been modified in [3, 4]. In particular, a modification of the criterion in order to reduce the effect of tracking problems is used in [3]. In [5], a modified MMSE detector, hereafter referred to as detector A, is proposed that improves the performances of the modified MMSE detectors in [3, 4], in a flat fading channel.

Detector A compensates for channel phase and amplitude variations by using a simple channel estimator whose input is derived from the output of an adaptive linear filter. The tap weights for this adaptive linear filter are determined by an orthogonal decomposition-based least mean square (LMS) algorithm. Pilot symbols are periodically inserted into the data stream to aid in channel estimation and algorithm adaptation.

The proposed improved detector, hereafter referred to as detector B, for the flat-fading channel uses the same basic structure as detector A, but a modified adaptive algorithm is used in detector B to improve the performance, stability, and versatility. This detector is then extended to the Rayleigh faded frequency-selective channel.

## II. System model

We consider a chip and bit asynchronous BPSK-modulated CDMA system. At first, we will consider the channel to be a Rayleigh flat-fading channel, with AWGN. There are $K$ users, and $(2M+1)$ bits in the data block transmitted by each user. The received signal can be written as

$$r(t) = \sum_{k=1}^{K} \sum_{m=-M}^{M} A_k b_k(m) c_k(m) s_k(t - mT - \tau_k) + \sigma n((t)) \quad (1)$$

where $A_k$ is the received amplitude of the $k$th user's signal, such that $A_k^2$ is the energy of the $k$th user; $b_k(m) \in \{-1,+1\}$ is the $m$th bit transmitted by the $k$th user; $c_k(m) = \alpha_k(m) e^{j\phi_k(m)}$ is the channel coefficient for user $k$, bit $m$; $s_k(t)$ is the deterministic signature waveform of the $k$th user, normalized to give unit energy; the discrete time representation of $s_k(t)$, designated by $\mathbf{s}_k$ is of length $PG$ chips per bit, where $PG$ is the processing gain; the individual chips of $\mathbf{s}_k$ have values from $\{-1/(\sqrt{PG}), 1/(\sqrt{PG})\}$; $\tau_k$ represents the relative transmission delays of the $k$th user; $n(t)$ is white Gaussian noise with unit power spectral density; the noise power within a frequency bandwidth, $B$, is $2\sigma^2 B$, with $N_o = 2\sigma^2$

A modified MMSE detector similar to that of [5] is illustrated in Figure 1. The soft decision output, $\hat{\mathbf{b}}_{sk}(m)$, for the bit estimate, $\hat{\mathbf{b}}_k(m)$, is used for the two-path RAKE receiver in Figure 2. The switch at the bottom right of Figure 1 is set to the "down" position when a pilot bit is expected in the received signal. The received signal, $r(t)$, is sampled once per chip to yield $\mathbf{r}_k(m)$, which is chip and bit aligned to user $k$.

The modified MMSE criterion is given by

$$E[|e_k(m)|^2] = E[|\hat{c}_k(m) d_k(m) - \mathbf{w}_k^H(m) \mathbf{r}_k(m)|^2], \quad (2)$$

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where $e_k(m)$ is the error signal; $d_k(m)$ is the data bit (pilot or bit estimate); $\mathbf{w}_k(m)$ is the tap weight vector for the adaptive filter, and $[\ ]^H$ indicates the Hermitian operation (complex conjugate and transpose); $\hat{c}_k(m)$ is the complex channel coefficient estimate.

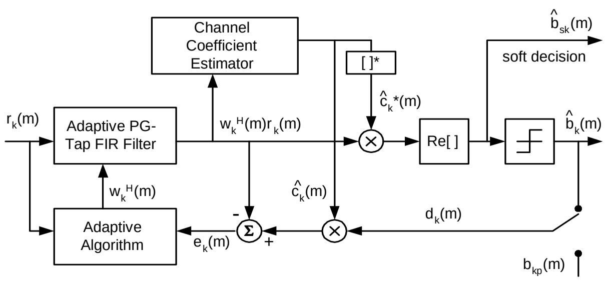


**Figure 1: Modified MMSE Detector**

Some detectors apply the channel equalization, using $\hat{c}_k^*(m)$, to the input of the detector, at $\mathbf{r}_k(m)$ [4, 6]. The detector considered here applies the channel equalization within the detector, as shown in Figure 1.

The tap weight vector is split into two separate orthogonal components, one of which is the user's signature and the other is the adaptive portion, $\mathbf{x}_k$, i.e.

$$\mathbf{w}_k(m) = \mathbf{s}_k + \mathbf{x}_k(m) . \qquad (3)$$

The only signals $\mathbf{w}_k(m)$ which satisfy (3) and the orthogonality of $\mathbf{x}_k(m)$ and $\mathbf{s}_k$ are those that satisfy the constraint

$$\mathbf{w}_k^H(m)\mathbf{s}_k = 1 , \qquad (4)$$

where $\mathbf{s}_k$ is the signature vector of user $k$ [1].

This tap weight vector is adapted by an orthogonal decomposition-based LMS algorithm,

$$\mathbf{x}_k(m+1) = \mathbf{x}_k(m) + \mu e^*(m)\mathbf{r}_x(m) , \qquad (5)$$

where [ ]* denotes the complex conjugate; μ is the step size of the algorithm; $\mathbf{r}_x$ is the projection of the received signal vector $\mathbf{r}_k$ on $\mathbf{x}_k$,

$$\mathbf{r}_x(m) = \mathbf{r}_k(m) - (\mathbf{r}_k^H(m)\mathbf{s}_k)\mathbf{s}_k . \qquad (6)$$

A simple channel estimator takes a moving average of pilot symbols, using the output of the adaptive filter. The pilot symbols are inserted every $P$ symbols, at which time the detector switches from a decision-directed mode, to a single-symbol training mode. The channel estimate remains constant between pilot symbols. At each pilot symbol, the channel estimate is updated using

$$\hat{c}_k(m) = \frac{1}{N_p}\sum_{i=0}^{N_p-1} b_{kp}(m-iP)\mathbf{w}_k^H(m-iP)\mathbf{r}_k(m-iP) , \qquad (7)$$
$$m \in \{\ldots,-2P,-P,0,P,2P,\ldots\} ,$$

where $N_p$ is the number of pilot symbols used, and $b_{kp}$ is the pilot symbol of user $k$.

## III. Rayleigh Flat Fading Receiver Improvements

The LMS algorithm in (5) requires knowledge of a fixed step size. If the number of users changes while the detector is in operation, the step size, μ, may no longer be optimum. To avoid the need to know the number of users in the channel, the normalized LMS (NLMS) algorithm can be used. The NLMS algorithm also exhibits a rate of convergence that is potentially faster than that of the LMS algorithm, due to the time-varying step-size parameter [7].

In the NLMS algorithm, (5) is modified to

$$\mathbf{x}_k(m+1) = \mathbf{x}_k(m) + \frac{\eta}{\varepsilon + |\mathbf{r}_k(m)|^2} e^*(m)\mathbf{r}_x(m) , \qquad (8)$$

where η is a constant step size, which controls the speed of convergence of the algorithm and ε is a small positive constant to ensure that if the input power, $|\mathbf{r}_k(m)|^2$, goes to zero, the algorithm will remain stable.

In practice, digital implementation of any algorithm results in performance degradation due to quantization errors. These errors accumulate and are shown in Section V to cause the components of the right hand side of (3) to become non-orthogonal. The component $\mathbf{x}_k(m+1)$ is orthogonally projected onto $\mathbf{s}_k$ with each update of the algorithm, so as to reduce problems due to quantization errors [8]. That is, after the tap weights are updated in (5) or (8), we apply

$$\mathbf{x}_k(m+1) = \mathbf{x}_k(m+1) - (\mathbf{x}_k(m+1)^T\mathbf{s}_k)\mathbf{s}_k . \qquad (9)$$

## IV. Rayleigh Frequency-Selective Fading Receiver

In a two path frequency-selective Rayleigh fading channel, the received signal is given by

$$r(t) = \sum_{k=1}^{K}\sum_{m=-M}^{M} (A_k b_k(m) c_{k,1}(m) s_k(t - mT - \tau_{k,1}) + A_k b_k(m) c_{k,2}(m) s_k(t - mT - \tau_{k,2})) + \sigma n(t) \qquad (10)$$

where $\tau_{k,1}$, $\tau_{k,2}$, $c_{k,1}$ and $c_{k,2}$ are the timing offsets and channel coefficients for the first and second paths of user $k$ respectively.

To cope with the frequency-selective fading channel, multiple instances of our "improved modified MMSE detector" can be used in a RAKE receiver structure. As an example, we can place two of these single-path detectors in parallel, using one detector for each of two paths. Such an approach using a different modified MMSE detector is used in [3]. Detector A is shown to have a lower BER than that in [3] for single path channels. Since detector B has a lower BER than that of detector A, it is expected that our multipath RAKE receiver (based on detector A) will have a better performance than that in [3].

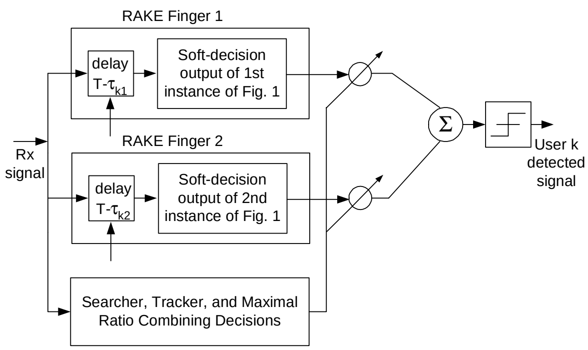


**Figure 2: RAKE Structure using the Improved Modified MMSE Detector**

The RAKE detector using the “improved modified MMSE detector” is shown in Figure 2. The “precombining” structure is the same as in [3]. The precombining structure means that the interference suppression (the detector of Figure 1) occurs before the multipath combining. The alternative is known as the “postcombining” structure, in which the interference suppression occurs after the multipath combining. It has been shown in [3] that the precombining structure is more effective for fast fading channels, using the modified MMSE criterion. The RAKE structure above uses maximal ratio combining to determine the weighting applied to the fingers before summing.

## V. SIMULATION RESULTS

Numerical results are obtained using Monte-Carlo simulation. In each trial of a simulation, 10,000 data bits were generated for each user, with the first 1000 bits to allow the tap weights to settle, and the last 9000 bits counting towards the final BER. The simulations of [5] calculate BER in steady state, after the tap weights have settled. This trial was repeated 500-5000 times, totaling 5,000,000 to 50,000,000 bits per data point in the plot below. All users have equal power, and undergo independent Rayleigh flat fading. The Rayleigh fading coefficients are regenerated with each simulation trial, using Clarke’s model [9]. The carrier frequency is 2 GHz, and the data rate is 3.968 Mcps. The mobile speed is 50 km/h, giving a Doppler frequency of about 92 Hz.

The $E_b/N_o$ is set at 20 dB. All users have independent random timing offsets making them completely asynchronous. Gold sequences of length 31 are used to spread each data bit. A randomly chosen Gold sequence was used for the desired user for each simulation trial. The pilot insertion times were chosen to be every 8 symbols. Pilot symbol averaging takes place over the current pilot symbol, plus the last two pilot symbols (3 symbols total). The optimum step size for the NLMS algorithm was found to be approximately 0.11.

Figure 3 shows the BER of detector B compared with the results for detector A reported in [5]. It was found, following a search for a better step size for the LMS algorithm, that the BER of detector A could be reduced substantially. This searching was performed using 15, 20, and 25 users. However, since the step size must be changed each time the number of users changes, this may pose a problem in practice.

Figure 3 also shows the simulation results when we find the best step size for 15 users, then keep that step size for 20 and 25 users (“Detector A - best step size for 15 users”). As shown in the plot, the performance degrades rapidly when the step size is not appropriate for the input received power. The plot also shows that even with the best step size for detector A, detector B still performs better. Detector B has a BER of 0.0127 at 15 users, with a 99% confidence interval of +/- 3%. The BER for detector A using the best step size is 0.0140, with a 99% confidence interval of +/- 7%.

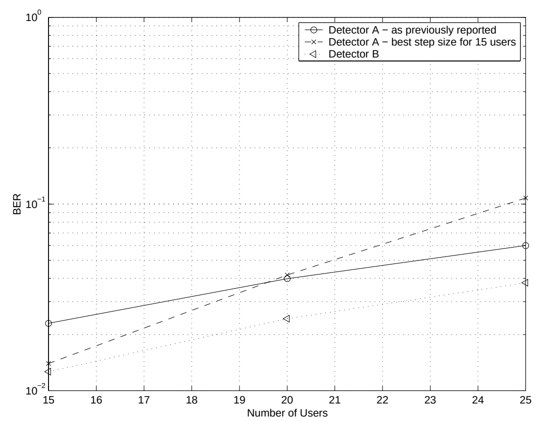


**Figure 3: BER of the Improved Modified MMSE Detector (detector B) vs. the Modified MMSE Detector of [5] (detector A)**

To illustrate the usefulness of the orthogonal projection in (9), the values of $\mathbf{x}_k{}^H\mathbf{s}_k$ are shown for a simulation of detector A without the orthogonal projection. Normally, $\mathbf{x}_k{}^H\mathbf{s}_k = 0$, because $\mathbf{x}_k$ and $\mathbf{s}_k$ are orthogonal components, and the orthogonal-decomposition based algorithm should ensure that orthogonality. But with the ‘right’ combination of quantization errors, the orthogonality is lost. As shown in Figure 4, for the first 100-200 bits or so, the orthogonality is preserved, but then the errors build up, and the components lose their orthogonality.

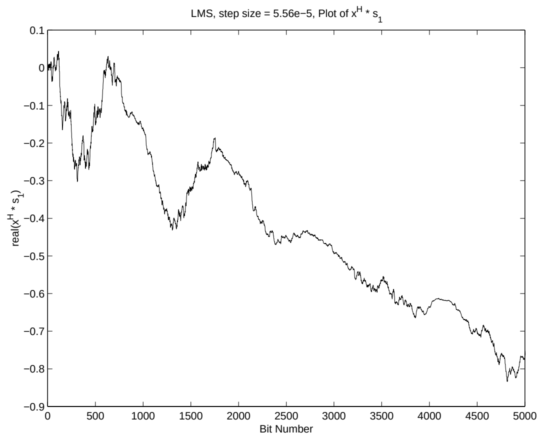


**Figure 4: Illustration of orthogonality problems without explicit orthogonal projection**

## VI. CONCLUSIONS

The performance and versatility of an existing modified MMSE detector were improved by modifying the adaptive algorithm. Firstly, the LMS algorithm was replaced by a normalized LMS algorithm. Secondly, a procedure was incorporated to ensure that the two components of the tap weight vector remain orthogonal.

A RAKE receiver structure to use this modified detector in a Rayleigh faded frequency-selective channel was also proposed. The simulations for the BER of the RAKE receiver are being finalized and results are not yet available.

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