Performance of MT-CDMA System with Post-Detection Diversity Reception over Rician Fading Channel

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Abstract **—This paper presents the theoretical analysis for Non-coherent multi-tone code division multiple access (MT-CDMA) system in terms of average bit error rate (BER). The system uses differential binary phase shift keying (DBPSK) modulation and post-detection diversity combining, considering slowly Rician fading and frequency selective channel in the indoor environment. Post-detection diversity is considered because it non-coherently combines the received signals from different diversity branches after differential detection. Here, the influence of the order of diversity and number of tones is studied for given bandwidth (BW), bit rate and transmitter power.**

Index terms **— Multi-tone, CDMA, Post-Detection and Diversity**

I. Introduction

MT-CDMA technique that combines Orthogonal Frequency Division Multiplexing (OFDM) and direct-sequence spread-spectrum (DS-SS) modulation has been proposed in [1]. This combination provides both high data rate transmission and multiple access capabilities. Over a frequency selective channel, an MT-CDMA signal experiences inter-symbol interference (ISI), inter-carrier interference (ICI) and multiple-access interference (MAI).

A lot of research has been performed on MT-CDMA system considering all the aforementioned interferences. Among those investigations we first considered [2] non-coherent demodulation, since, synchronous carrier recovery is a difficult task in a fading multipath environment. In [2] we studied the performance of MT-CDMA system using DBPSK modulation for multipath Rician fading channel. To improve the average BER encountered in [2], here, we are introducing post-detection diversity combining for the same system.

As in [2], we consider the situation in which the receiver can acquire time synchronization with the desired signal but not phase synchronization, and hence the modulation scheme of interest is Differential Phase Shift Keying (DPSK). Post-detection diversity weights and combines all branches after signal detection and do not require the difficult-to-implement co-phasing function. All the techniques used with pre-detection diversity such as maximal-ratio combining (MRC), equal-gain combining (EGC) and selection combining (SC) can be applied [3] to post-detection diversity.

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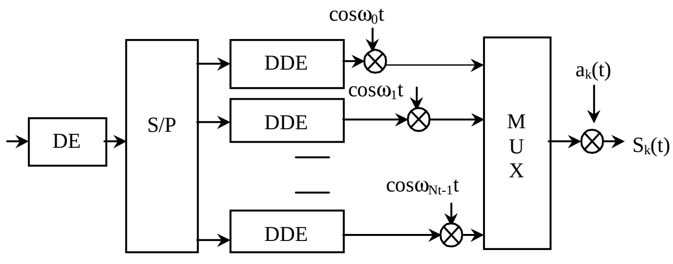

Fig.1. Block diagram of Non-coherent MT-CDMA transmitter for user k [DE → Data encoder, S/P → Serial to Parallel converter, DDE → Differential data encoder]

The investigation by Dennis Lee et al. [4] shows that post-detection MRC is less complex than the post-detection EGC while the former performs better in terms of BER over both post-detection SC and EGC. Adachi and Ohno [5] drew the same conclusions. All these conclusions have led us to present the analysis for the non-coherent MT-CDMA system with post-detection MRC, using DBPSK.

The structure of the system is first presented in Section II. In Section III, the Bit Error Probability of this system is analyzed for a multipath Rician fading channel in the presence of MAI in an indoor environment. Section IV provides some numerical results. Finally conclusions are drawn.

II. System Model

A. Transmitter

Fig.1 shows the block diagram of the MT-CDMA transmitter for the link of *kth* user ($1 \le k \le K$). Here, the BPSK-symbol stream at the rate N_t/T is first split into N_t parallel streams with symbol duration T. These symbols are differentially encoded in each branch. The *pth* encoded symbol stream modulates a tone f_p. The carriers are orthogonal on the symbol duration and hence are given by $f_p = f_0 + p/T$ where f_0 is the RF signal.

The multitone signal is obtained by adding different carriers. Spectrum spreading is achieved by multiplying the multitone signal with the pseudo-noise (PN) sequence associated with the user under consideration. This PN sequence is denoted by $a_k(t)$ and has a chip duration of $T_c = T/N_c$ where N_c is the sequence length. The sequence $a_k(t)$ is periodic with period N_c. The signal transmitted by user k can be written as

$$S_k(t) = \sqrt{2P} \sum_{p=0}^{N_t-1} \mathrm{Re}\left[a_k(t) b_{p,k}(t) \exp\left(2\pi j f_p t + j\theta_{p,k}\right)\right] \qquad (1)$$

where, $a_k(t)$ is given by

$$a_k(t) = \sum_{n=0}^{N_c-1} a_k^n P_{T_c}(t - nT_c), \tag{2}$$

$b_{pk}(t)$ is the differentially encoded binary data symbol associated with *pth* tone of user k and f_p is the *pth* carrier frequency. P is the power associated with all users.

We assume a perfect power control mechanism, so that the system is not affected by the near-far problem. $\theta_{p,k}$ is a constant phase angle introduced by the modulator of *kth* user using *pth* tone. In equation (2), $a^n{}_k \in \{1, -1\}$, is the *nth* bit in the PN-sequence and P_{Tc} is a rectangular pulse of duration T_c.

B. Channel

The channel between user k's transmitter and receiver is characterized by the low-pass equivalent impulse response function of the form:

$$h_k(t) = \sum_{l=1}^{L} \beta_{kl} \exp(j\phi_{kl})\delta(t - \tau_{kl}) \tag{3}$$

In (3), kl refers to path l of user k with total number of paths L+1, while ϕ_{kl} and τ_{kl} are the phases and the propagation delays respectively. A commonly used assumption is γ_{kl} and τ_{kl} are independent and uniformly distributed where $\gamma_{kl} \in [0, 2\pi]$ and $\tau_{kl} \in [0, T/N_t]$. Finally, for the slow fading channel we can assume that the phase angles γ_{kl} and $\theta_{p,k}$ with other random parameters associated with the channel do not vary significantly over the duration of two adjacent data symbols. The path gains β_{kl}s' are Rician variables, and are assumed to be identically distributed and independent for different values of k and l. The Rice *probability density function* (pdf) is given by

$$d_\beta(r) = \frac{r}{\sigma_r^2} \exp\left[-\frac{r^2+S^2}{2\sigma_r^2}\right] I_0\left(\frac{rS}{\sigma_r^2}\right) \tag{4}$$

where, $r \geq 0$, $S \geq 0$. The Rician parameter, $R = S^2/2\sigma_r^2$, represents the ratio of the power associated with the direct component and the scattered component.

C. Receiver

1) *The Received Signal*: The block diagram of the receiver with diversity combiner for *user 1* is shown in Fig. 2 for a rectangular chip waveform. Here, we use matched filter-based receiver, which is sub-optimum for the channel under consideration. The signal received at any given receiver is:

$$r(t) = \sqrt{2P} \sum_{k=1}^{K} \sum_{l=1}^{L} \sum_{p=0}^{N_t-1} \beta_{kl} a_k(t - \tau_{kl}) b_{pk}(t - \tau_{kl}) \cos(2\pi f_p t + \phi_{pkl}) + n(t) \tag{5}$$

Here, $\phi_{pkl} = (\theta_{pk} - 2\pi f_p \tau_{kl} + \gamma_{kl})$ is uniformly distributed (0 to 2π) random phase and $n(t)$ is the Additive White Gaussian Noise (AWGN) of (two sided) spectral density $N_0/2$.

2) *Matched Filter Output:* Let $g_x(t)$ and $g_y(t)$ be the in-phase and the quadrature components of the matched filter output in the receiver. Now choosing arbitrarily, *user* 1 as the reference user using *qth* tone, we have at $t=T$, the nominal sampling point,

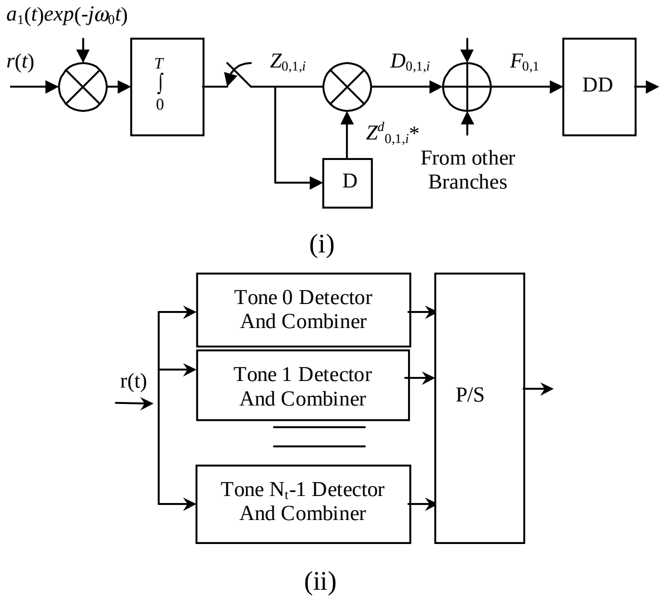

Fig.2. Block diagrams of the receiver for user 1 i) Tone-0 detector and combiner ii) The complete receiver [D → Symbol-Delay block, DD →Decision Device, P/S → Parallel to Serial data converter]

$$g_x(T) = \int_0^T r(t) a_1(t) \cos(2\pi f_q t) dt \tag{6a}$$

$$g_y(T) = \int_0^T r(t) a_1(t) \sin(2\pi f_q t) dt \tag{6b}$$

Now let us assume that the 1*st path* between the transmitter of *user* 1 and the corresponding receiver is the reference path and all other paths and tones constitute interference. So without loss of generality, we can write $\tau_{11} = 0$ and $\phi_{q11} = 0$.

Using these values and the periodicity of the code waveforms, upon omitting the terms with $2f_0$, equations, (6a) and (6b) become:

$$\begin{aligned} g_x(T) &= \sqrt{P/2}\beta_{11} b_{q1}^0 T + \sqrt{P/2}\left(b_{q1}^{-1} Y_{qq,1}^{cc} + b_{q1}^0 \hat{Y}_{qq,1}^{cc}\right) \\ &+ \sqrt{P/2} \sum_{p=0 \neq q}^{N_t-1} \left\{ b_{p1}^{-1}\left(X_{pq,1}^{cc} - X_{pq,1}^{ss}\right) + b_{p1}^0\left(\hat{X}_{pq,1}^{cc} - \hat{X}_{pq,1}^{ss}\right)\right\} \\ &+ \sqrt{P/2} \sum_{k=2}^{K} \sum_{p=0}^{N_t-1} \left\{ b_{pk}^{-1}\left(X_{pq,k}^{cc} - X_{pq,k}^{ss}\right) + b_{pk}^0\left(\hat{X}_{pq,k}^{cc} - \hat{X}_{pq,k}^{ss}\right)\right\} \\ &+ \eta_{q1} \end{aligned} \tag{7a}$$

$$\begin{aligned} g_y(T) = -\Big[&\sqrt{P/2}\left(b_{q1}^{-1} Y_{qq,1}^{sc} + b_{q1}^0 \hat{Y}_{qq,1}^{sc}\right) + \sqrt{P/2} \sum_{p=0 \neq q}^{N_t-1} \left\{ b_{p1}^{-1}\left(X_{pq,1}^{cs} + X_{pq,1}^{sc}\right)\right. \\ &\left. + b_{p1}^0\left(\hat{X}_{pq,1}^{cs} + \hat{X}_{pq,1}^{sc}\right)\right\} + \sqrt{P/2} \sum_{k=2}^{K} \sum_{p=0}^{N_t-1} \left\{ b_{pk}^{-1}\left(X_{pq,k}^{cs} + X_{pq,k}^{sc}\right)\right. \\ &\left. + b_{pk}^0\left(\hat{X}_{pq,k}^{cs} + \hat{X}_{pq,k}^{sc}\right)\right\} - \psi_{q1}\Big] \end{aligned} \tag{7b}$$

where $b^{-1}{}_{pkx}$ and $b^0{}_{pkx}$ denote the previous and current bits respectively, of user k, transmitted by tone p and, η and ψ terms are the noise samples of independent zero-mean random variables with identical variance $N_0T/4$.

In equation (7),

$$Y_{qq,k}^{xy} = \sum_{l=2}^{L} g_x(\phi_{qkl}) R_{qq,k}^y(\tau_{kl}) \ \ \& \ \ \hat{Y}_{qq,k}^{xy} = \sum_{l=2}^{L} g_x(\phi_{qkl}) \hat{R}_{qq,k}^y(\tau_{kl}) \tag{8a}$$

$$X^{xy}_{pq,k} = \sum_{l=1}^{L} g_x(\phi_{pkl}) R^{y}_{pq,k}(\tau_{kl}) \ \ \& \ \ \hat{X}^{xy}_{pq,k} = \sum_{l=1}^{L} g_x(\phi_{pkl}) \hat{R}^{y}_{pq,k}(\tau_{kl}) \quad (8b)$$

$$g_x(\phi_{qkl}) = \beta_{kl} f(\phi_{qkl}) \quad (8c)$$

where, the partial correlation terms (R-terms) are given by,

$$R^{x}_{pq,k}(\tau_{lk}) = \int_{0}^{\tau_{kl}} a_1(t) a_k(t-\tau_{kl}) f\{2\pi(p-q)t/T\} dt \quad (8d)$$

$$\hat{R}^{x}_{pq,k}(\tau_{lk}) = \int_{\tau_{kl}}^{T} a_1(t) a_k(t-\tau_{kl}) f\{2\pi(p-q)t/T\} dt \quad (8e)$$

In equations (8c-8e), $f(.)$ corresponds, either to a *sine*-function ($x = s$) or to a *cosine*-function ($x = c$). Equation (7a) can be analyzed as follows: The first term represents the desired component. The term with $k = 1$ and $p = q$ is the ISI due to multipath effect. The terms with $k = 1$ and $p \neq q$ are the interference due to the loss of orthogonality between the different carriers at different frequencies. The term with $k \neq 1$ is associated with MAI. The final term is the AWGN.

Now, let $Z_{q,1}$ and $Z^{d}_{q,1}$ denote the complex envelopes of the matched filter output at the current and the previous sampling instants respectively i.e.

$$Z_{q,1} = g_x(T) + jg_y(T) \quad (9a)$$

$$Z^{d}_{q,1} = g^{d}_{x}(T) + jg^{d}_{y}(T) \quad (9b)$$

Since we have considered slow fading channel environment compared to the bit-rate in use, we can assume that $Z^{d}_{q,1}$ differs from $Z_{q,1}$, only in the data bits and in the AWGN samples involved. Based on this assumption we get another pair of equations for (9b), same as (7a) and (7b) where, b^{0}_{pk} and b^{-1}_{pk} are replaced by b^{-1}_{pk} and b^{-2}_{pk} respectively, while γ_{q1} and ψ_{q1} are replaced by γ^{d}_{q1} and ψ^{d}_{q1} respectively. In this case, the data bit b^{-2}_{pk} is transmitted two-bit interval prior to b^{0}_{pk}.

3) *Differential Detection and Post-detection Diversity combining:* The output of the differential detector at the sampling instant is given by:

$$D_{q,1} = \mathrm{Re}\left(Z_{q,1} Z^{d}_{q,1}{}^{*}\right) \quad (10)$$

where, * denotes complex conjugation. When there is no diversity, $D_{q,1}$ is the decision variable. In case of post-detection diversity, assuming that the fading signals at the different diversity branches are independent and have identical statistical characteristics, the desired decision variable for the *ith* order of diversity is given [5] by:

$$F_{q,1} = \sum_{i=1}^{M} D_{q,1,i} \quad (11)$$

where, M is the total number of diversity branches.

III. Performance Evaluation: Bit Error Probability

To evaluate the performance of post-detection combining, we assume that all interferences are Gaussian distributed. With this assumption, the sum of interference and noise, in $Z_{q,1}$ and $Z^{d}_{q,1}$ denoted by $N_{q,1}$ and $N^{d}_{q,1}$ respectively, are also Gaussian. The decision variable $F_{q,1}$ is then given by:

$$F_{q,1} = \mathrm{Re}\left[\sum_{i=1}^{M}\left(\sqrt{P/2}\,\beta_{11,i} b^{0}_{q1} T + N_{q,1,i}\right)\left(\sqrt{P/2}\,\beta_{11,i} b^{-1}_{q1} T + N^{d}_{q,1,i}\right)^{*}\right] \quad (12)$$

where, $\beta_{11,i}$ denotes the gain and $N_{q,1,i}$ and $N^{d}_{q,1,i}$ are the Gaussian random variables associated with the *ith* path. It can easily be shown that the variances of $N_{q,1,i}$ and $N^{d}_{q,1,i}$ and covariance between them are all independent of i. Now we will assume that for each i the noise variable $N_{q,1,i}$ is independent of $N^{d}_{q,1,i}$. This assumption is reasonable because the $cov[N_{q,1,i}, N^{d}_{q,1,i}{}^{*}]$ is negligibly small compared to $var[N_{q,1,i}]$ and $var[N^{d}_{q,1,i}{}^{*}]$.

We will also assume that the pair $(N_{q,1,i}, N^{d}_{q,1,i})$ is independent of the pair $(N_{q,1,i}, N^{d}_{q,1,j})$ for $i \neq j$. Since in this case, each set of delays is taken with reference to a different time origin (corresponding to the arrival time of the signal in the corresponding combined path) and also any two resolved paths (i, j) are separated by at least a chip time period, the assumption is physically reasonable. Letting equal average path power for all the paths and users in the channel under consideration and, with all the assumptions mentioned above, the bit error probability of DBPSK based non-coherent MT-CDMA system can be derived [6,7] as:

$$\begin{aligned} P_2(e) = {} & Q(a,b) - I_o(ab)\exp\left(-\frac{a^2+b^2}{2}\right) \\ & + 2^{1-2M} I_o(ab)\exp\left(-\frac{a^2+b^2}{2}\right)\sum_{i=0}^{M-1}\binom{2M-1}{i} \\ & + 2^{1-2M}\exp\left(-\frac{a^2+b^2}{2}\right)\sum_{n=1}^{M-1} I_n(ab) \\ & \left\{\sum_{i=0}^{M-1-n}\binom{2M-1}{i}\left[\left(\frac{b}{a}\right)^{n} - \left(\frac{a}{b}\right)^{n}\right]\right\} \end{aligned} \quad , M \geq 2 \quad (13a)$$

and

$$P_2(e) = Q(a,b) - \frac{I_o(ab)}{2}\exp\left(-\frac{a^2+b^2}{2}\right), \qquad M = 1 \quad (13b)$$

where $I_n(x)$ is the *nth* order modified Bessel function of first kind, $a = 0$, $b =$ ***sqrt***$(2\gamma_b)$ and

$$Q(a,b) = \exp\left(-\frac{a^2+b^2}{2}\right)\sum_{m=0}^{\infty}\left(\frac{a}{b}\right)^{m} I_m(ab), \qquad b > a > 0 \quad (14)$$

The quantity γ_b is the total received average *signal to noise ratio* (SNR), which is given by $\gamma_b = A/d$ where,

$$A = \sum_{i=1}^{M} \beta^{2}_{11i}\left(S^2 + 2\sigma^2\right)^{-1} \quad (15)$$

$$d = \left\{\left[\frac{1}{T^2}\sum_{p=0\neq q}^{N_t-1}\left[\left\{E\left[\left(R^{c}_{pq}\right)^2\right]\right\} + \left\{E\left[\left(R^{s}_{pq}\right)^2\right]\right\}\right]\right. \right. \left. + \frac{1}{3N_c N_t}\right]2(KL-1) + \left. \frac{N_0}{\bar{E}_s}\right\} \quad (16)$$

In equation (16) the subscript k has been omitted from the correlation terms because the expected values of the squared correlation terms are independent of user k [1]. In the same equation, $\bar{E}_s = E_s(S^2+2\sigma^2)$ is the mean received symbol energy and $E_s = PT$ is the received symbol energy.

The probability of error given by (13) can be considered as a conditional error probability for given γ_b and denoted by $P(e|\gamma_b)$. So, the probability of bit error $P_2(e)$ for DBPSK based non-coherent MT-CDMA system for a particular user using a particular tone can be obtained as:

$$P_2(e) = \int_0^{\infty} P(e \mid \gamma_b)\, p(A)\, dA \quad (17)$$

where, the $p(A)$, the pdf of A can easily be shown as:

$$p(A) = (R+1)\left\{\frac{A(R+1)}{MR}\right\}^{\frac{M-1}{2}} \exp[-\{MR + A(R+1)\}] \quad (18)$$
$$I_{M-1}\left\{\sqrt{AMR(R+1)}\right\}$$

IV. Numerical Results and Discussion

Here, we present some numerical results in terms of BER. In all calculations, the Rician parameter R is set to 2 and a channel with 4 paths has been used. The maximum number of tones, used here is 32. The overall BW is considered to be constant. Since the constant BW represents identical chip duration, so the ratio between number of chips and number of tones is kept constant. We have kept a constant delay range irrespective of the number of tones, which is equal to the symbol duration corresponding to one-tone transmission. From the interference noise computation, it is verified that the central frequency is mostly affected by the cross carrier frequency with position $q = N_t/2$. This corresponds each time the worst case. In all the results we have considered this case.

Fig 3 shows the bit error probability versus the averaged received symbol energy over white noise ratio obtained for the diversity of order of M=1, 2, 4. There is no interfering user. The curves clearly show the positive effect of increasing the diversity order, as well as the number of tones.

Fig 4 shows the BER in the presence of multiple users. Here, we consider 10 users. The performance shows that even in the presence of MAI, a gain in the bit error probability can be achieved compared to one-tone situation. Here the performance is worse than the single-user system performance due to the presence of MAI. The curves show irreducible error probability at higher SNR. The irreducible error probability, in the interference-limited system, is the key performance indicator in indoor communications.

Both of the performance curves in figures 3 and 4 show higher diversity gain for higher number of tones. Comparison with the results of [2] shows that diversity reception provides significant reduction of error rate compared to non-diversity reception.

V. Conclusions

This paper presents analytical results for determining the performance of MT-CDMA communications system with DBPSK modulation and post-detection diversity for multipath Rician-fading channel. The study considers indoor environment. The investigation shows that, under a constrained bit-rate, the system provides two-fold gain: one is from the higher number of tones and the other is from diversity. Our future investigation will concentrate on the study of finding out an optimal diversity combining technique for non-coherent MT-CDMA system.

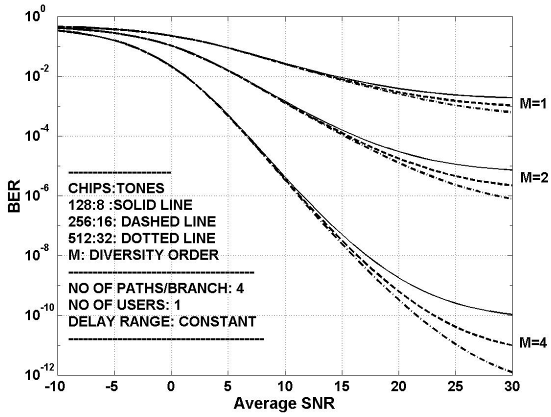

Fig 3. Comparison of bit error probability for various combinations of tones and PN sequence-length (without MAI)

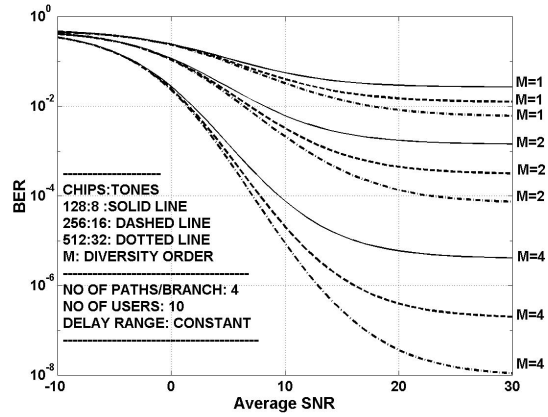

Fig 4. Comparison of bit error probability for various combinations of tones and PN sequence-length (with MAI)

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